Model Invariance Testing Under Different Levels of Invariance

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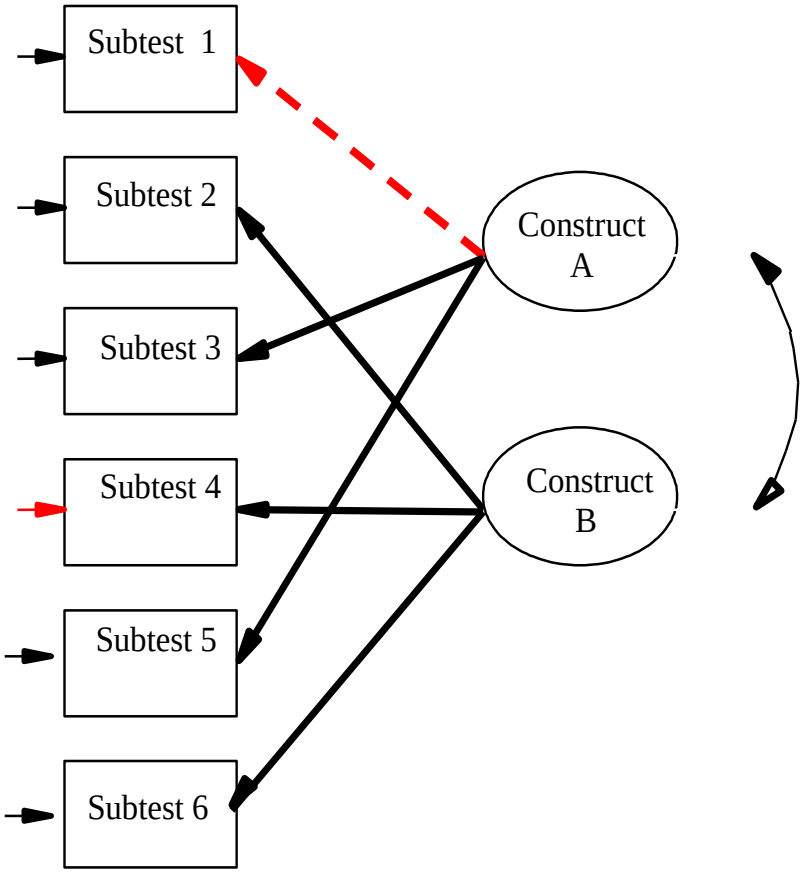
Measurement Invariance (MI)

- MI is important.
  - Construct comparability across groups cannot be assumed
  - Must be demonstrated with the proper empirical evidence
    - Wu, Li, & Zumbo (2007)
  - If MI assumption is not met the following crucial and commonly asked question may not be accurately answered:
    - Are group ability differences real or due to measurement problems? (e.g., Rock et al., 1978)

Measurement Invariance: Factor Invariance

- Within MI assessment, non-invariance of factor structure is a concern
- Multi-group confirmatory factor analysis (MCFA) is a popular method for comparing an instrument's *a priori* latent structure across groups or time
  - The ability to compare specific model features at the matrix level, as well as individual matrix elements.

Test Level Invariance
Factor Structure Invariance
Multisample Confirmatory Factor Analysis & Latent Mean Structures



- Analysis Steps
  - Multisample Confirmatory Factor Analysis
    - Test:
      - » Factor loadings (Λ)
      - » Error variances (θ)
      - » Factor variances / covariances (Φ)
    - Unconstrained vs. constrained matrices, compare difference in chi-square for decline in fit and or fit indices
    - If there is a significant decline, test individual components of matrix to locate invariance

- Analysis Steps
  - Latent Mean Structures Analysis
    - Are there differences on the latent variable means (ξ) between groups?
  - Testing requires:
    - Invariance of factor structure
    - Invariance of intercepts (T)
      - (Intercepts are examinees' predicted subtest scores)
  - Partial invariance can be permitted

Factor Invariance Research

- Simulation studies have examined the effectiveness of methods for assessing metric or factor loading invariance under various conditions
    - e.g. Kim, Kwok, & Yoon, 2012; French & Finch, 2006; 2011; Meade & Lautenschlager, 2004, Millsap & Olivera-Aguilar, 2012; Yang-Wallentin, Jöreskog & Luo, 2010 and a host of others

- Little investigation of the effectiveness of invariance testing approaches for other model parameters under various conditions after establishing the extent to which metric invariance holds.

MCFA and Invariance testing

- Performance of change in a goodness-of-fit index (Δ*CFI*) to detect a lack of MI remains unclear
- The chi-square difference test is used most
- Other *GFI*s have been advocated: Δ*CFI* is noteworthy
    - Commonly reported, readily available
    - Results have been mixed, only evaluated with ideal conditions or real data
        - e.g., Cheung & Rensvold, 2002; French & Finch, 2006; Wu et al., 2007

Research questions

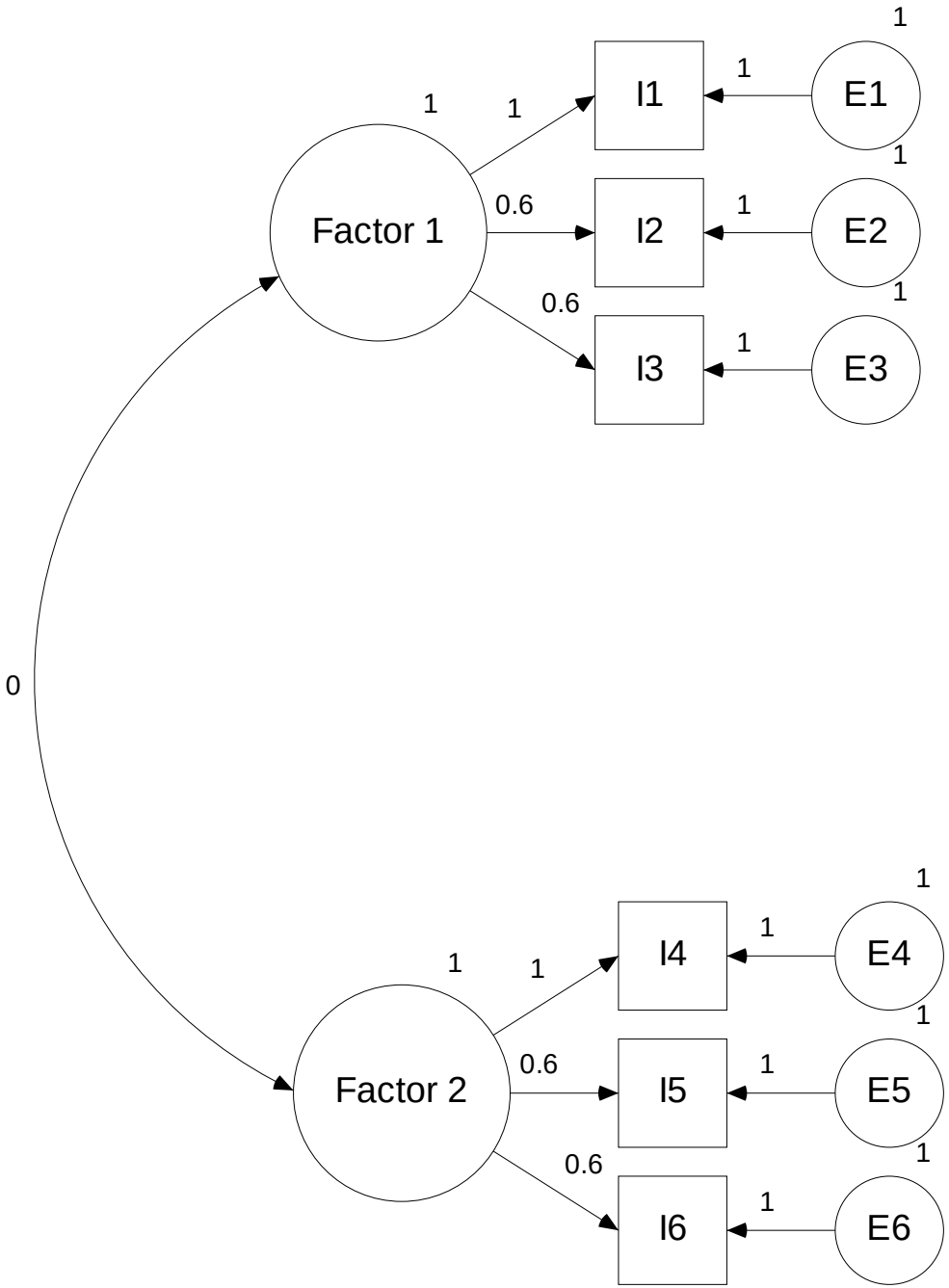
1. What are the Type I error and power rates for detection of intercept, factor covariance and variance, error variance and factor mean differences using the Chi-square difference test and the CFI difference?

2. Partial invariance: What is the impact of factor loading noninvariance on invariance testing of other model parameters?

Method

Monte Carlo simulation study (1000 replications)

Two groups



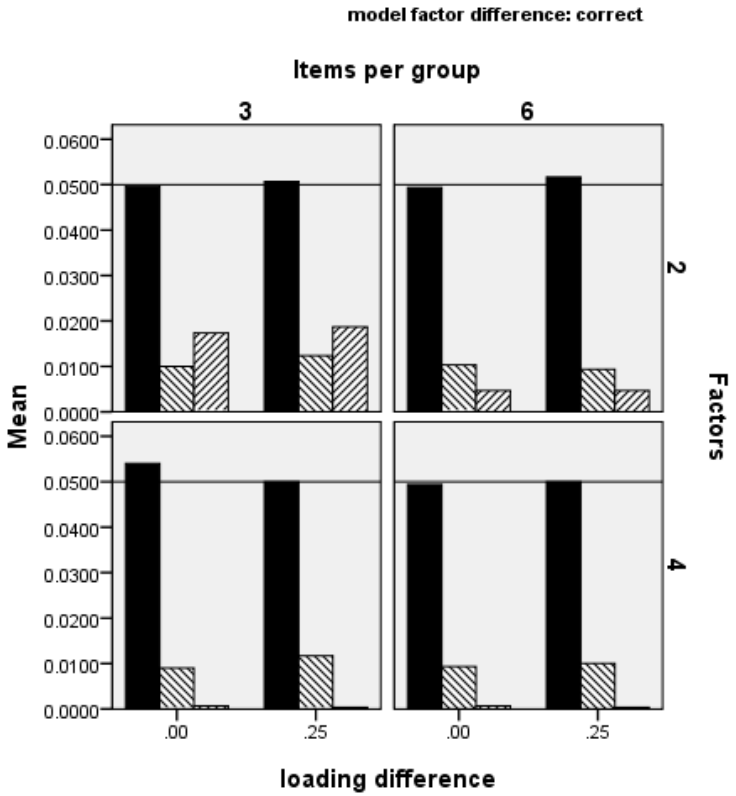
Method

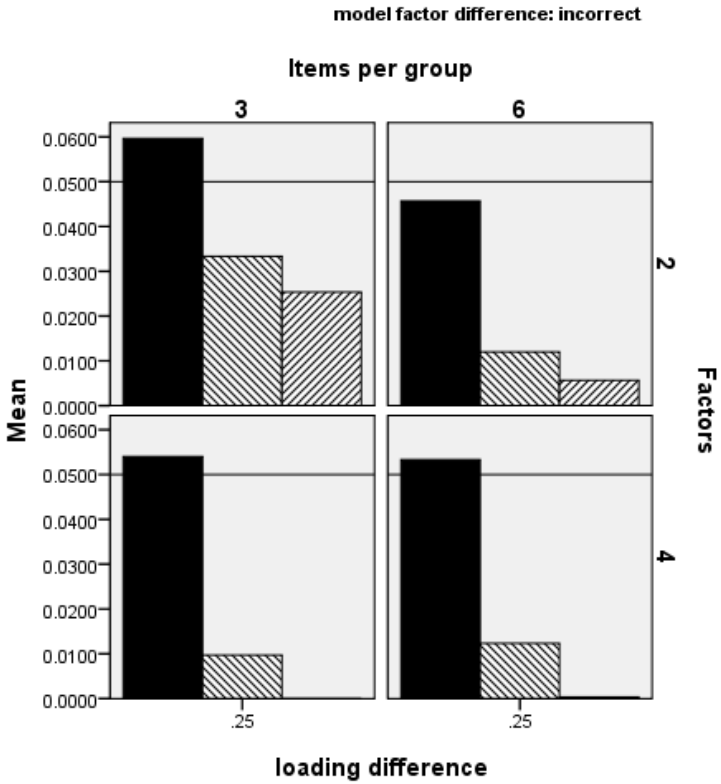
- Model parameter group differences:
  - Indicator intercept: 0, 0.2, 0.5, 0.8
  - Factor mean: 0, 0.2, 0.5, 0.8
  - Factor covariances (standardized): 0, 0.1, 0.3, 0.5
  - Factor variances: 0, 0.33, 0.66, 1.00
  - Error variances: 0, 0.33, 0.66, 1.00

Method

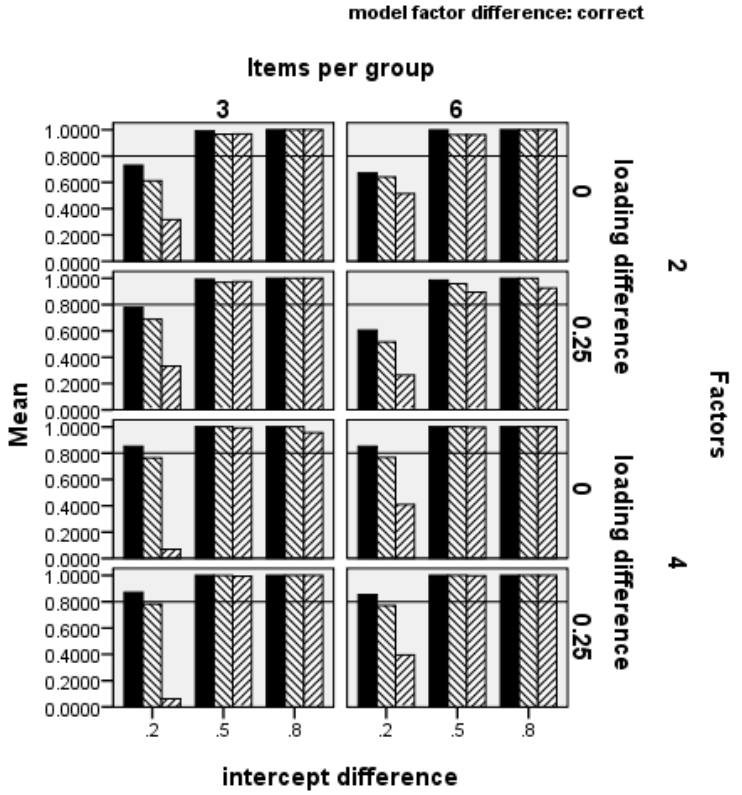
- Manipulated study conditions:
  - Sample size: 150/150, 500/500, 1000/1000
  - Number of factors: 2, 4
  - Number of indicators per factor: 3, 6
  - Proportion of model parameter noninvariance: 0, 0.34
  - Proportion of factor loading noninvariance: 0, 0.34
  - Modeling of factor loading noninvariance: Correct, incorrect
  - Factor loading difference: 0, 0.25
- Outcomes of interest:
  - Type I error rate
  - Power

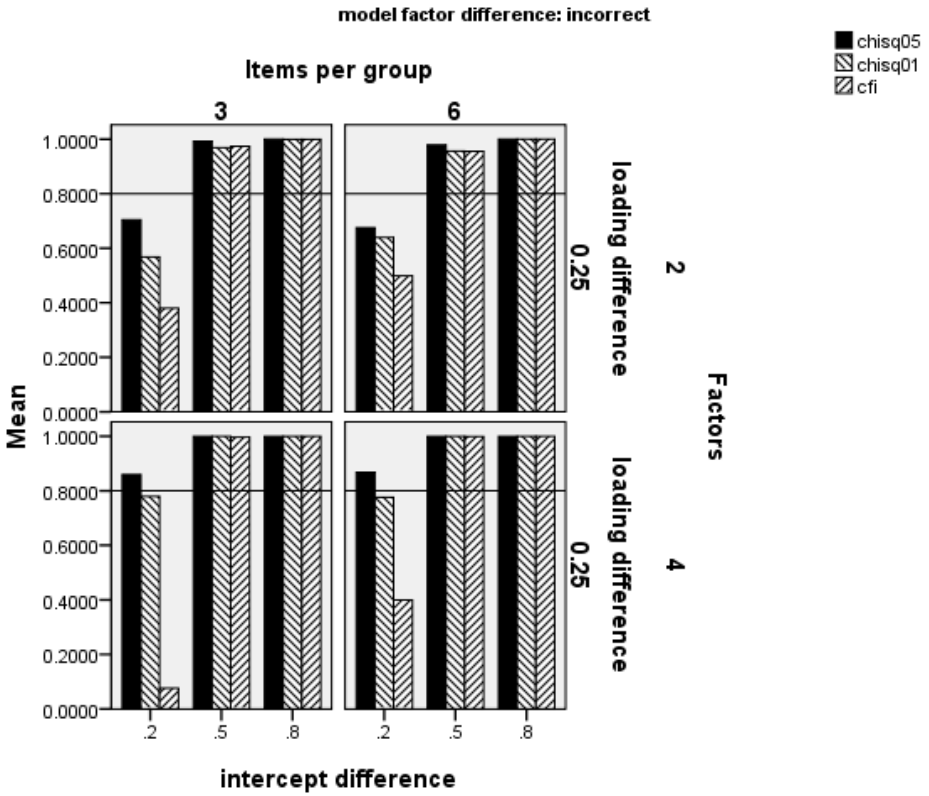
Intercept Invariance Type I error



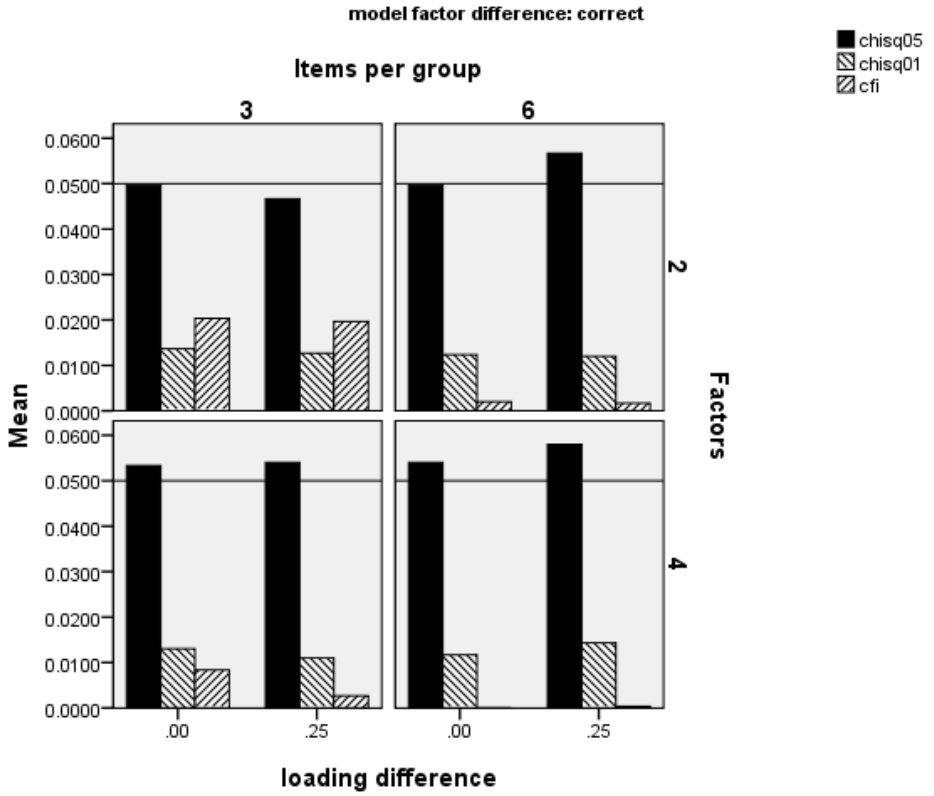


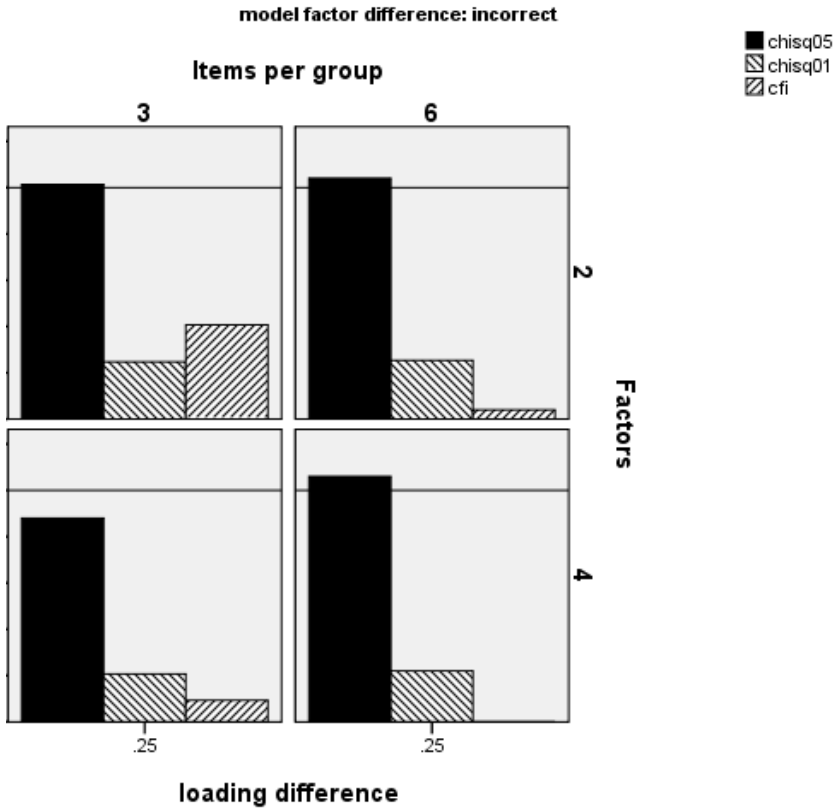
Intercept Invariance Power



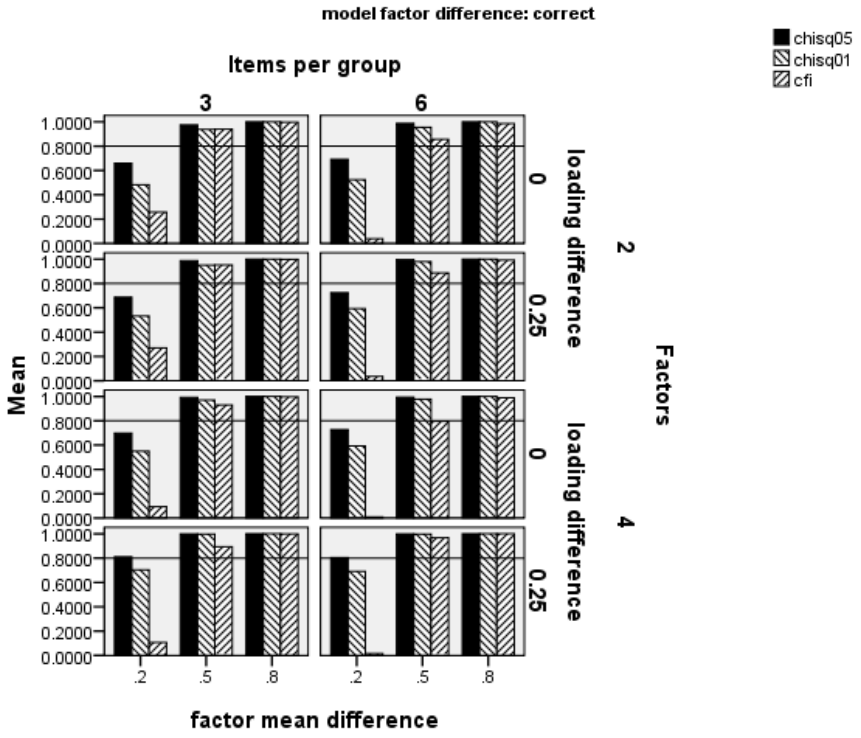


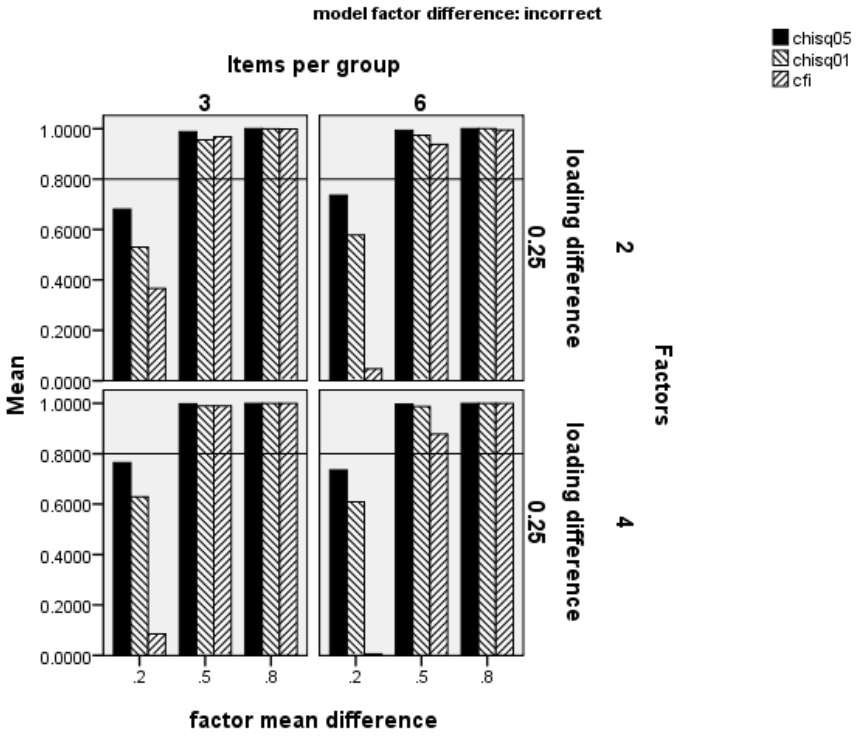
Mean Difference Type I error



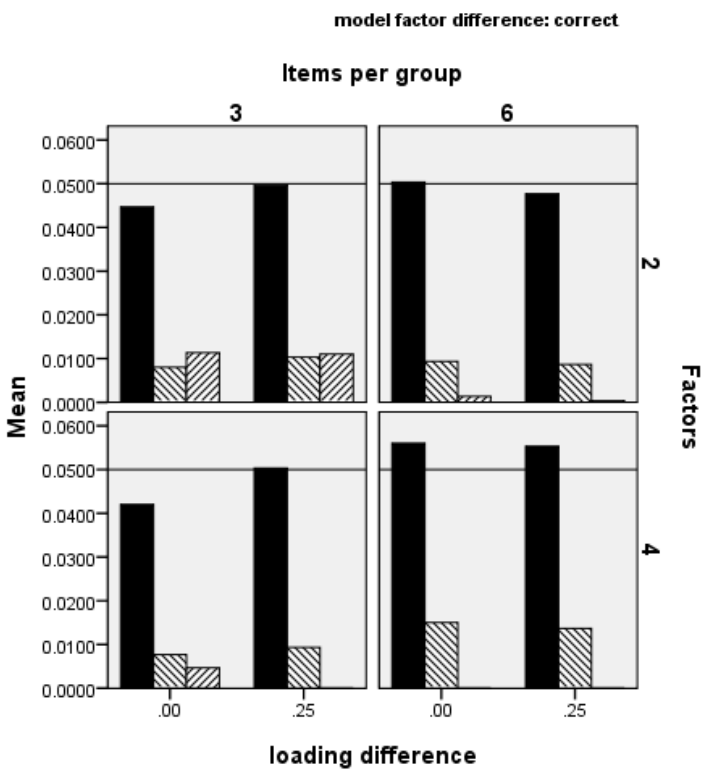


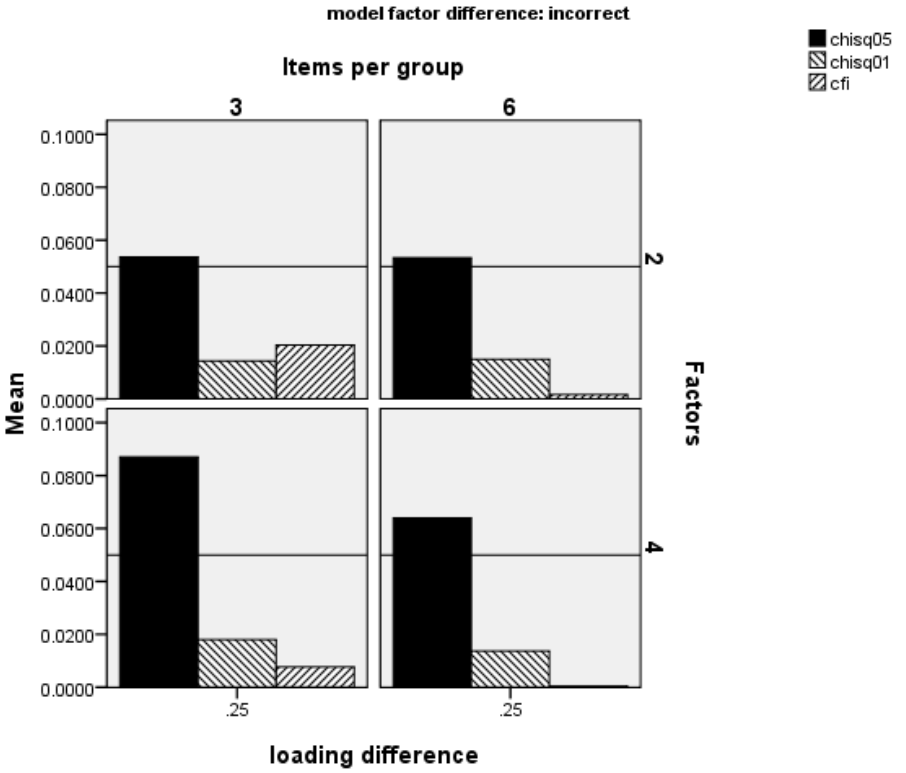
Mean difference Power



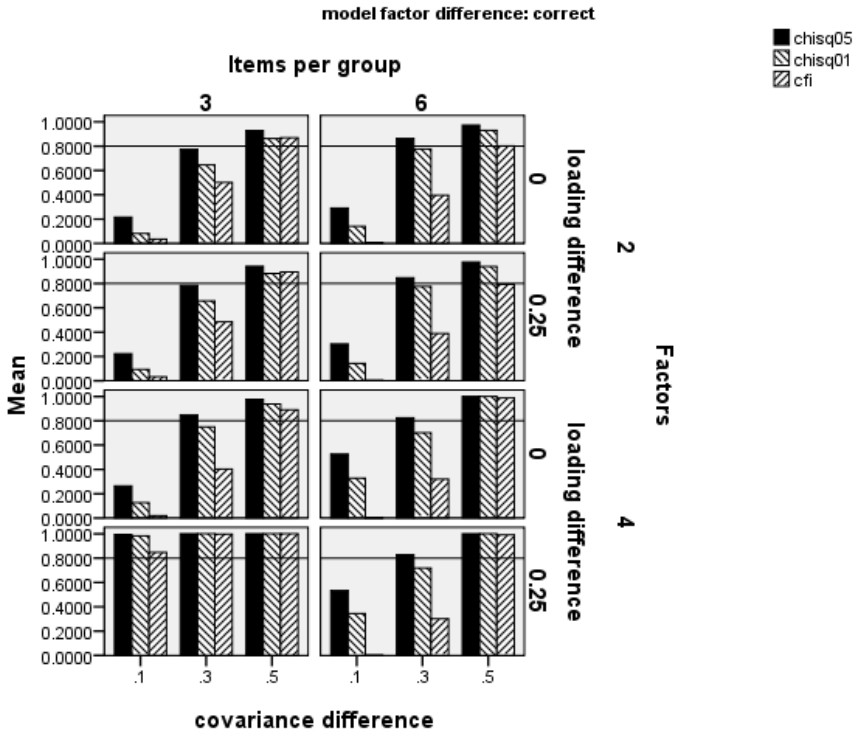


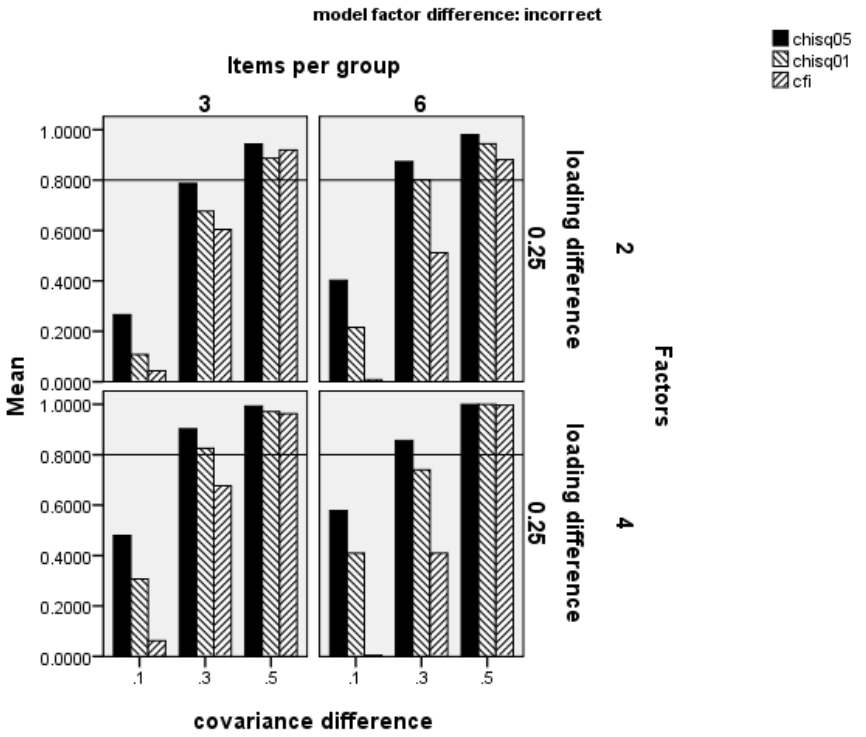
Factor Covariance Invariance
Type I error



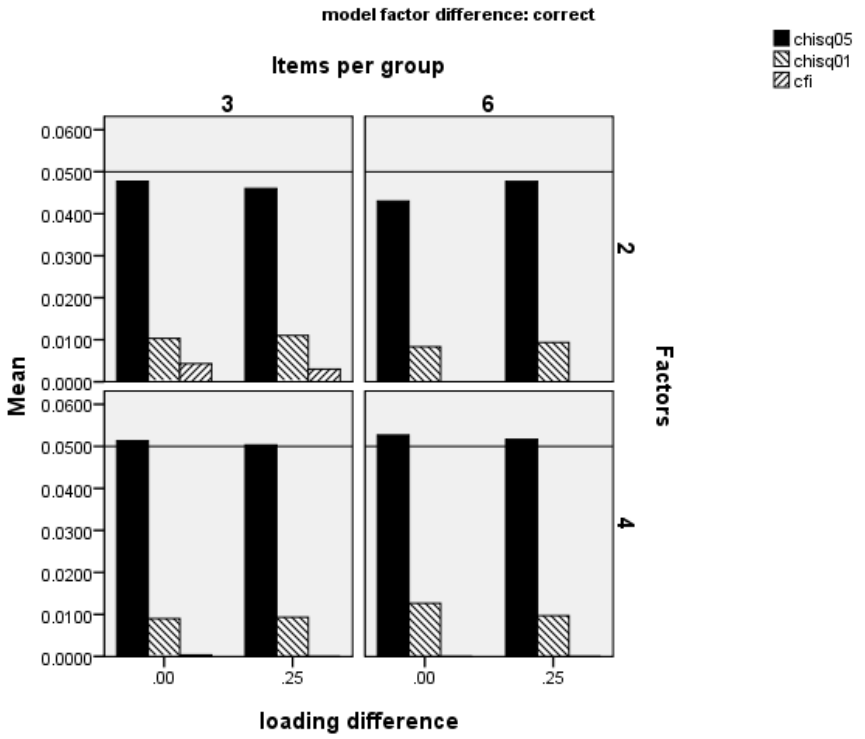


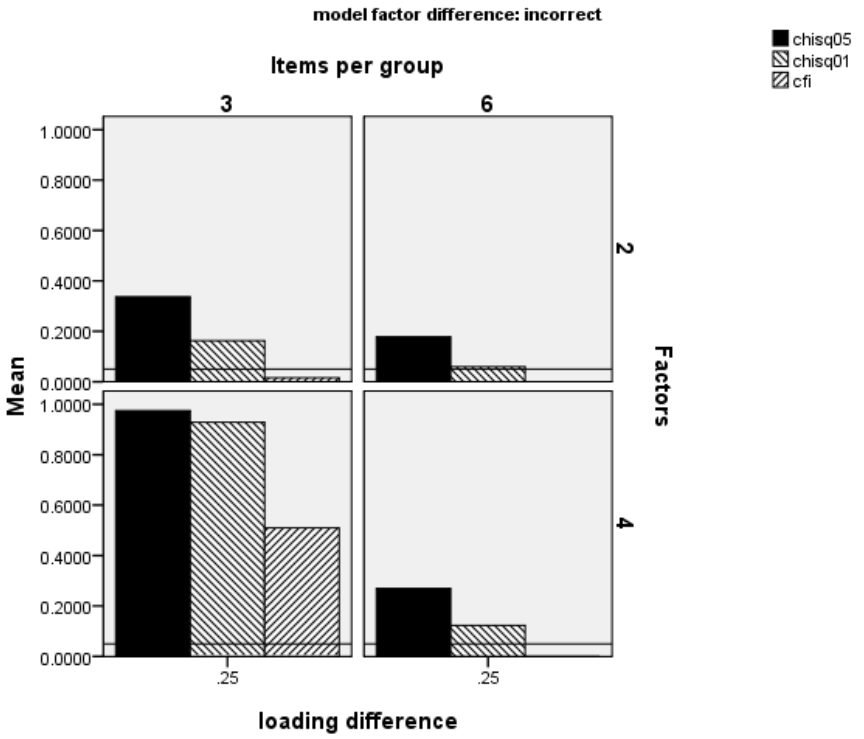
Factor Covariance Invariance Power



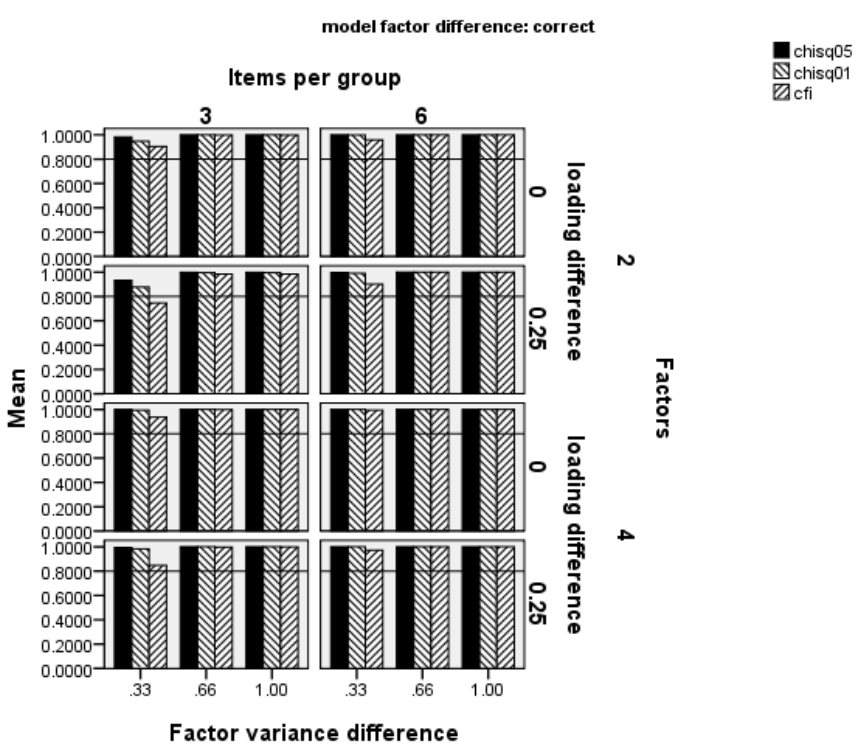


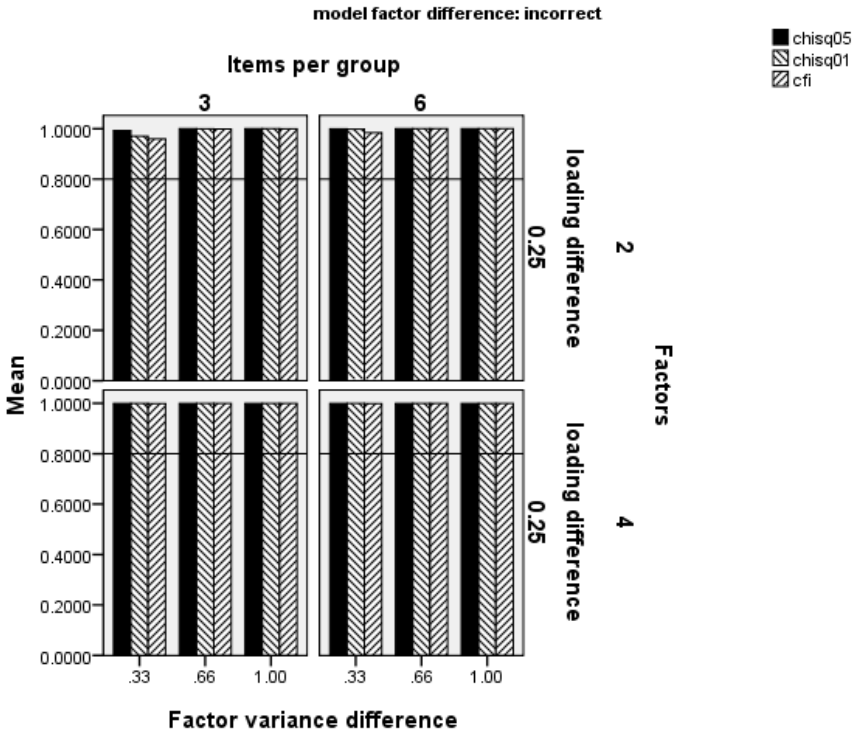
Factor Variance Invariance
Type I error



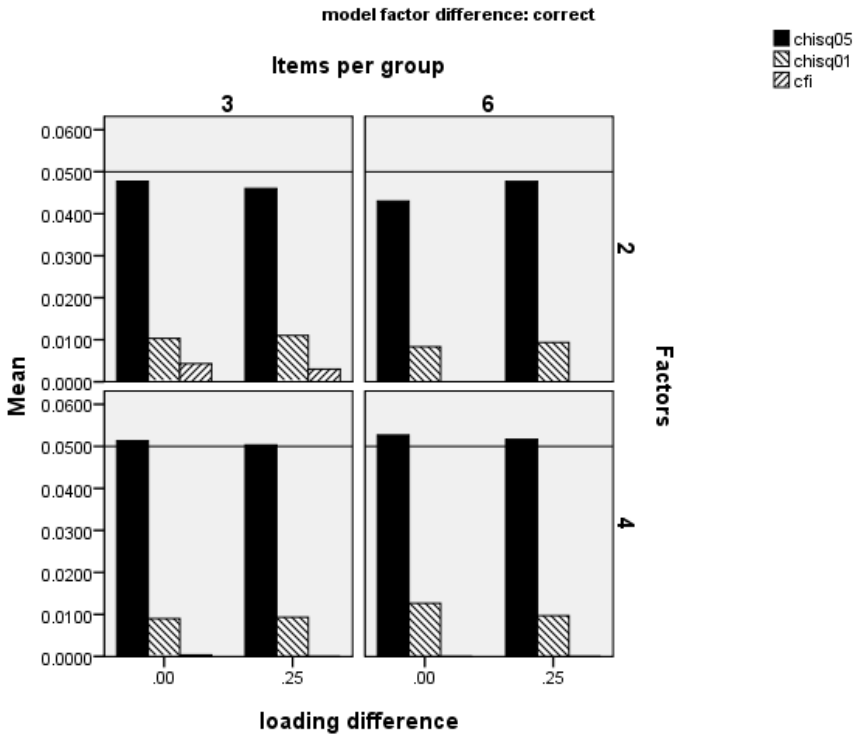


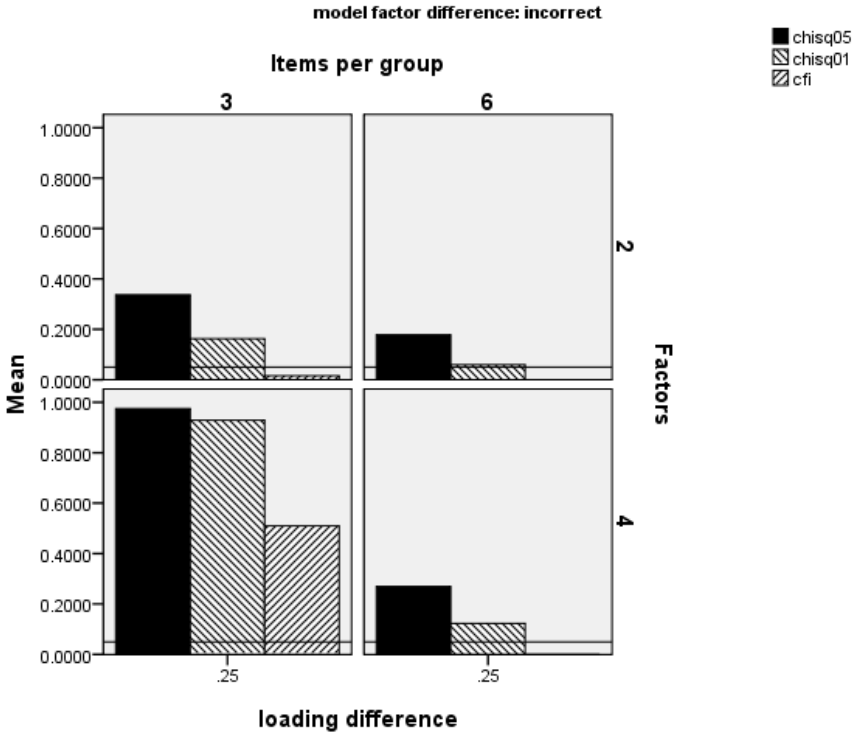
Factor Variance Invariance Power



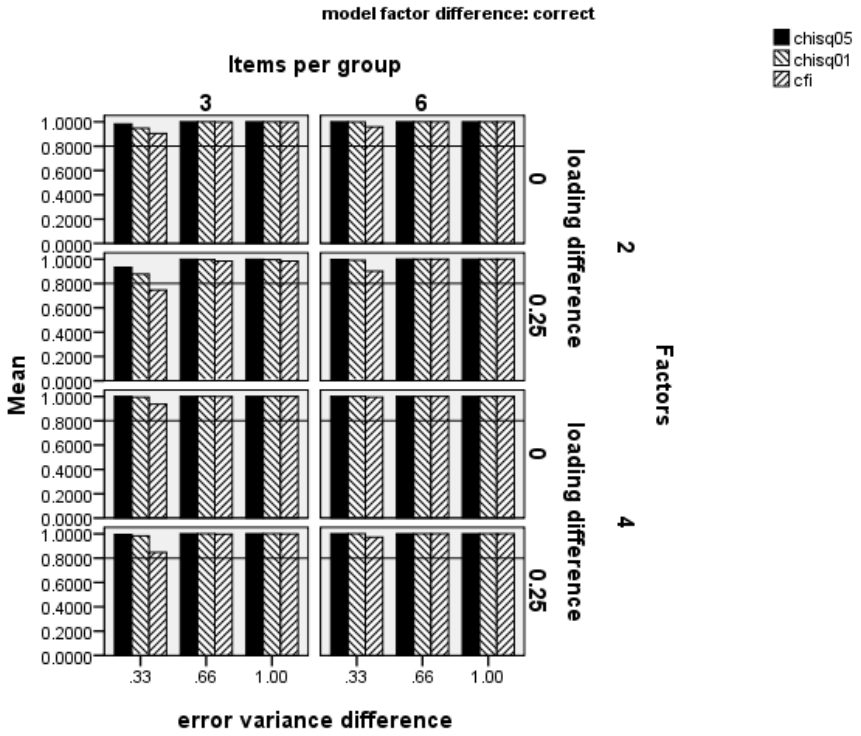


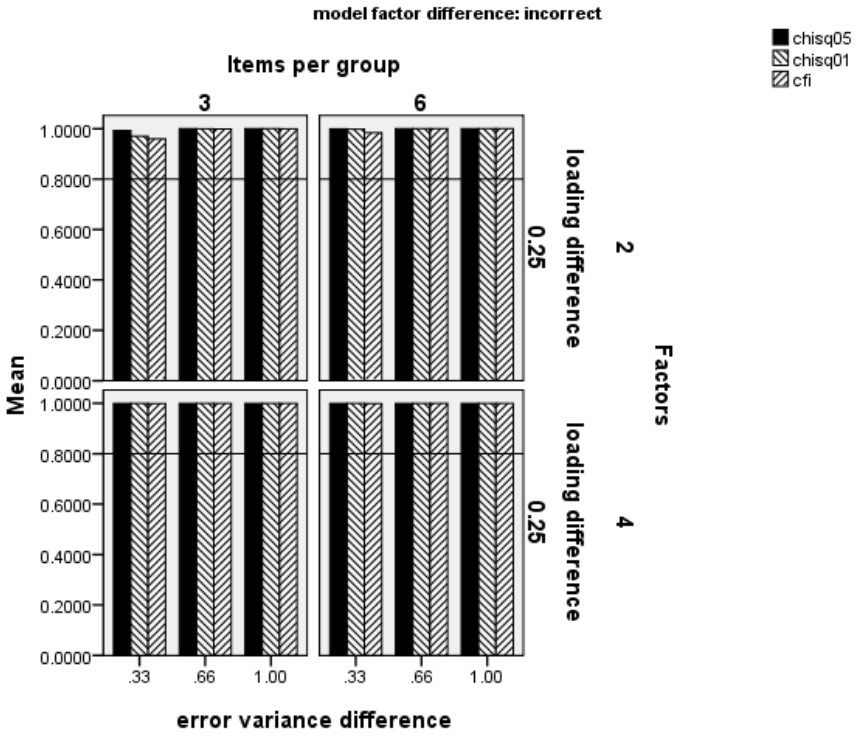
Error Variance Invariance Type I error



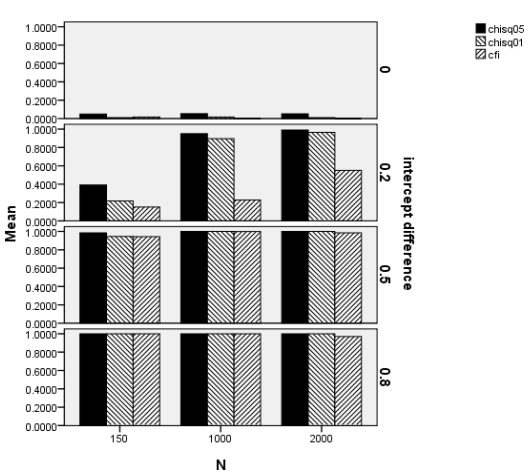


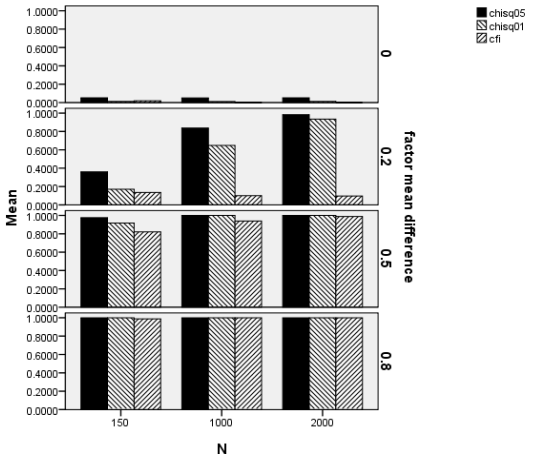
Error Variance Invariance Power

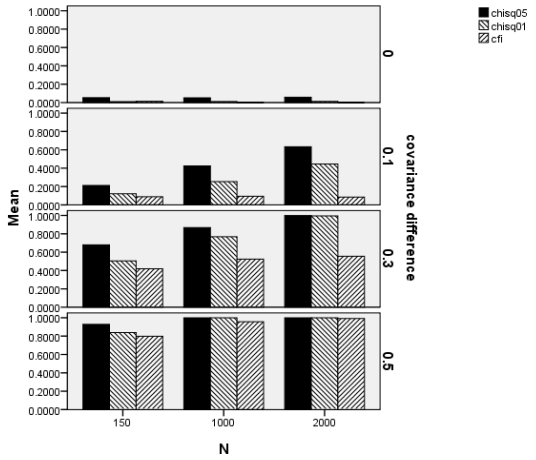


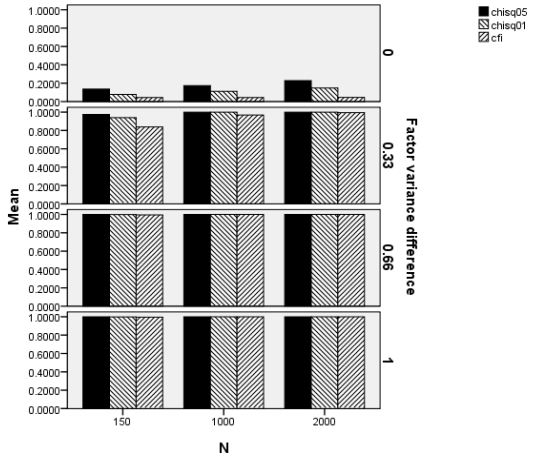


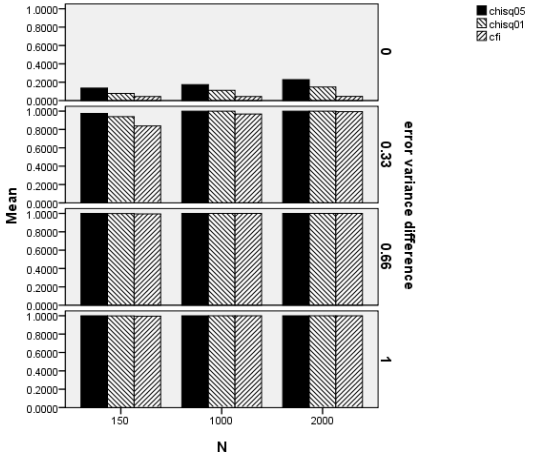
Rejection rates by level of group difference and sample size











Conclusions

- When factor loadings were invariant, Type I error for testing invariance of other model parameters was controlled.
- When factor loadings were noninvariant and modeled correctly, Type I error for testing invariance of other model parameters was controlled.
- When factor loadings were noninvariant and *not* modeled correctly, Type I error rates for testing the invariance of factor variance, and error variance were greatly inflated.
- When factor loadings were noninvariant and *not* modeled correctly, Type I error rates for testing invariance of factor covariances were inflated for 4 factors and 3 indicators per factor.

Conclusions

- Power rates were comparable across factor loading invariance conditions for all model parameters.

- Power rates were comparable across numbers of factors and indicators

Conclusions

- Researchers should carefully assess factor loading invariance prior to testing invariance of other model parameters.

- When modeled correctly, partial loading invariance has minimal impact on invariance assessment of other model parameters.

- When ignored, partial loading invariance will lead to inflated Type I error rates when testing the invariance of factor variance and error variance, but not other model parameters.

Future directions

- Investigate a greater variety of factor loading noninvariance conditions.
- Examine parameter estimates under the factor loading invariant conditions.
- Examine more fit indices and levels of change