Impact of Measurement Error in Continuous Time

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Question

Continuous time differential equations typically include a term associated with the stochastic innovations, or process error. If a single time series is analyzed that includes more than one type of error, how are estimates affected? Will a model that smoothes over the continuous time process provide better estimates than a model that tries to separate the process from stochastic error?

Modeling in Continuous Time

- Invariant to Time
- Independent of Time
- Two Sources of Error

- It is assumed that there is one, underlying process that is being estimated.
- Results in discrete time are dependent on length of time between measurements.
- Continuous time model results are independent of the lag.
- Measurement error
- Stochastic error – aka Brownian model, Wiener process, drunken walk in continuous time

Project Overview

Cross-lag panel models are popular in psychology for examining the relationship between two variables, and two possible methods for estimating those models in continuous time are Latent Differential Equations (LDE) and the Exact Discrete Model (EDM).

- LDE, similar to growth curve modeling, uses multiple observations to estimate derivatives, thereby producing an effect similar to a smoothing filter. Smoothing filters are often adept at addressing measurement error.
- EDM is a mathematically exact solution to the integration of a differential equation model with stochastic error in the process.

When a time series consisting of only a single measurement at each occasion is collected, the observations may represent a combination of process and measurement error; LDE and EDM are differentially well suited to addressing each type of error.

Impact of Measurement Error in Continuous Time

The matrix of auto-regressive (X1 to X2, Y1 to Y2) and cross-lag values (X1 to Y2, Y1 to X2) relate to the continuous time auto-effects (XX and YY) and cross-effects (XY and YX) through this formula:

\[ \mathbf{A}(\mathbf{M}_t) = e^{\mathbf{A}(\mathbf{M})} \]

where A is the matrix of auto-regressive and cross-lag values for some length of time t, e to the A is 2.718281 raised to A.

Tested A-matrix Values

<table>
<thead>
<tr>
<th></th>
<th>Discrete</th>
<th>Continuous</th>
</tr>
</thead>
<tbody>
<tr>
<td>XX</td>
<td>0.95, 0.90, 0.85, 0.80</td>
<td>-0.3307 to 0.01943</td>
</tr>
<tr>
<td>XY</td>
<td>-0.30, -0.10, 0.10, 0.30</td>
<td>0.4104 to 0.4104</td>
</tr>
<tr>
<td>YX</td>
<td>-0.40, -0.20, 0.00, 0.20, 0.40</td>
<td>-0.5472 to 0.5472</td>
</tr>
<tr>
<td>YY</td>
<td>0.92, 0.87, 0.82, 0.77</td>
<td>-0.37111 to -0.01132</td>
</tr>
</tbody>
</table>

1280 Simulation Conditions

- R (version 3.0.2) was used to simulate & analyze 1000 data sets per condition
- Four sample sizes: 50, 150, 250, 500
- Five levels of measurement error added to each data set: 0%, 10%, 15%, 20%, 25%
- Discrete-time panel model values selected for A matrix; log of matrix values calculated for each combination
- OpenMx (Boker et al., 2014) used to for both LDE and EDM
- CT_SEM.R with oversampling utilized for EDM (Vloeckle & Oud, 2013)
- Informative starting values provided for LDE and EDM

Results

- As expected, EDM provides the most accurate estimates when there is little or no measurement error.
- Auto-effect results decrease (corresponding to smaller estimates of discrete-time autoregressive effects) linearly with the addition of measurement error in EDM but remain steady with LDE.
- Cross-effect results increase linearly with the addition of measurement error in EDM but remain steady with LDE.
- EDM converged for every model while LDE failed to converge 0.45% of the time. Most convergence problems happened in models with no measurement error.
- Results do not differ based on sample size

Conclusion

- If measurement error ranging from 0-10% can be assumed, EDM will provide equally good or better estimates than LDE over a wide range of parameter values.
- If measurement error is greater than 10%, LDE should provide researchers with more accurate results over a wide range of parameter values.

Works Cited


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