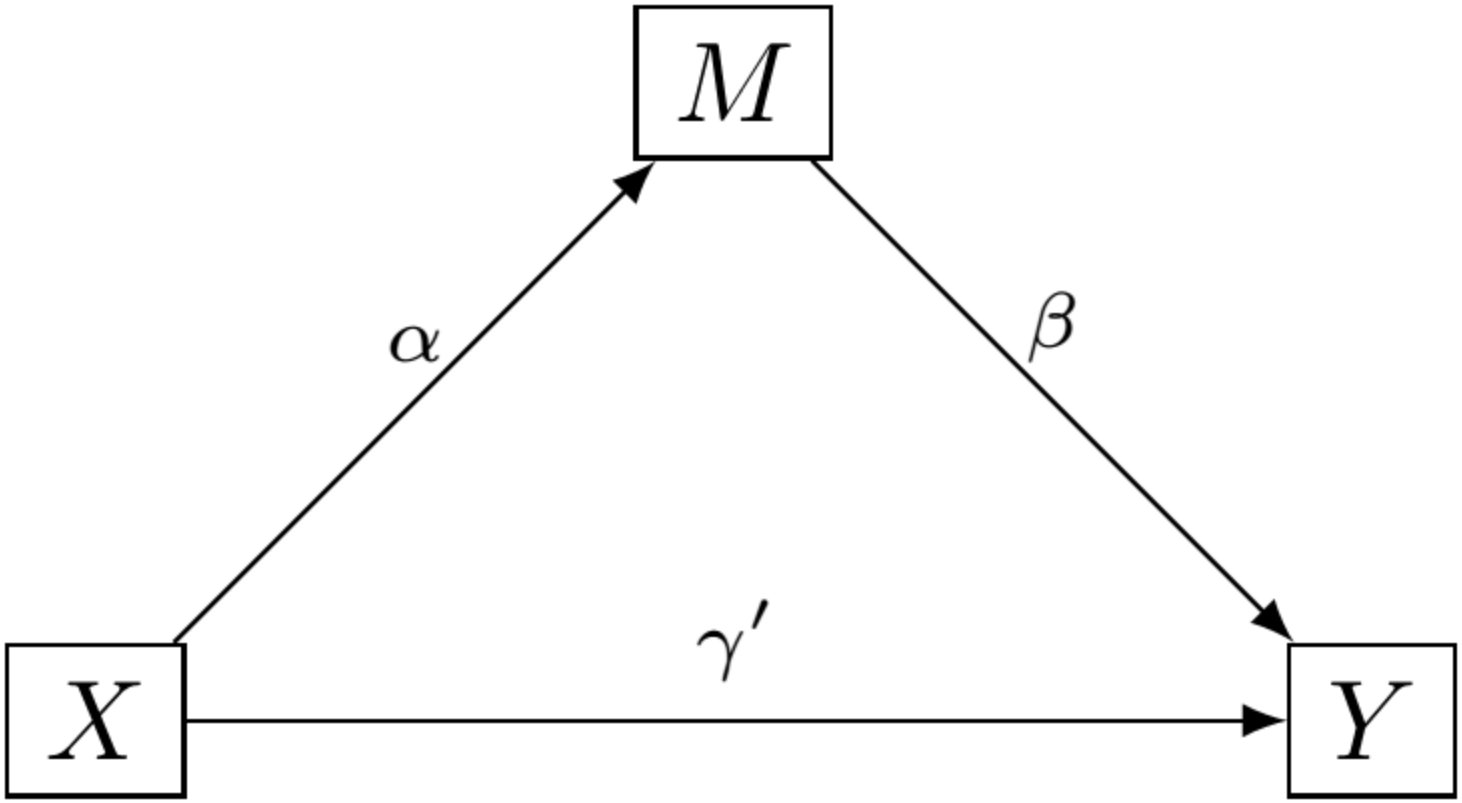
The Markov equivalence class of the mediation model

Felix Thoemmes, Wang Liao, & Marina Yamasaki
Cornell University

Mediation model



Mediation model

If M is a mediator, then indirect effect should be not equal to 0 (significant, given sufficient statistical power)

~~Significant indirect effect is proof that M is a mediator~~

~~Reversing / manipulating the arrows (testing the opposite model) proofs that one model is more plausible than the other~~

MacKinnon, D. P., Krull, J. L., & Lockwood, C. M. (2000). Equivalence of the mediation, confounding and suppression effect. *Prevention Science*, *1*(4), 173-181.

Fiedler, K., Schott, M., & Meiser, T. (2011). What mediation analysis can (not) do. *Journal of Experimental Social Psychology*, *47*(6), 1231-1236.

Equivalence class

- All models that are observationally equivalent are part of an equivalence class
- They all have the same implied covariance matrix

Equivalence class

- The equivalence class is an important concept, because it tells us precisely which other causal models have the exact same support from the data

- Using statistics (e.g., fit indices) alone does not provide *any* argument as to which model is preferred

Equivalence class

- *Assuming no confounding*, there are 27 just-identified alternative models that are all equally plausible, from statistical evidence alone

Same constraints on covariance matrix
Same model-implied covariance matrix
Same global fit indices
Different path coefficients

Equivalence class

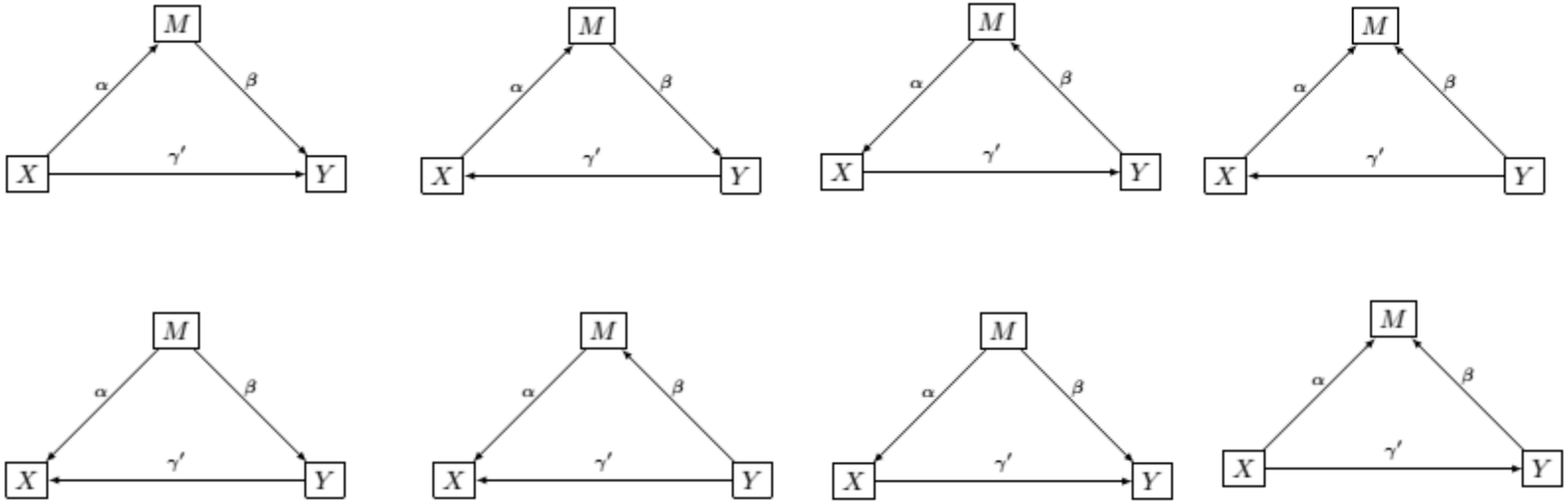
- This means that every time a researcher endorses a mediation model based on e.g., fit, or significance of an indirect effect, that researcher at the same time also endorses all other models in the equivalence class

Equivalence class

- Equivalent (Markovian) models can be found using substitution rules (Stelzl, 1986; Lee & Hershberger, 1990; Meek, 1995)

A path X→Y can be replaced with Y ← X if and only if, all "parents" of X are shared by Y

- In the mediation model case, this yields 8 equivalent models


M
α
β
X
γ′
Y

Equivalence class

- Semi-Markovian models (models that include bi-directed arrows) also have substitution rules (even though they are incomplete; Pearl, 2010; Desjardins, 2013)

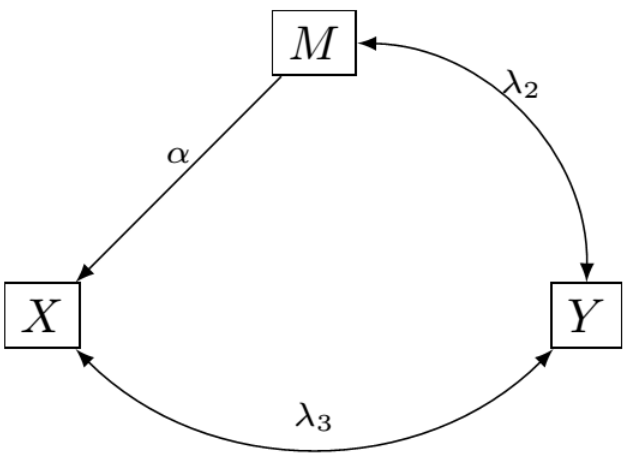
Equivalence class

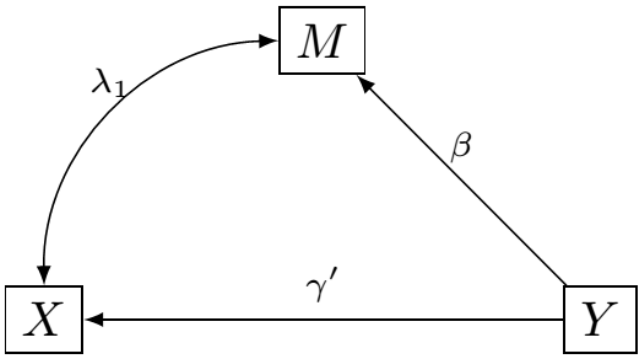
A path X→Y can be replaced with X ←→ Y if every neighbor (variable connected with bi-directed arrow) of X is a neighbor of Y, and every parent of X is a parent of Y

X → Y can be reversed into X ← Y if every neighbor of Y is a child of X, every neighbor of X is a child of Y, and every parent of X is a parent of Y

Equivalence class

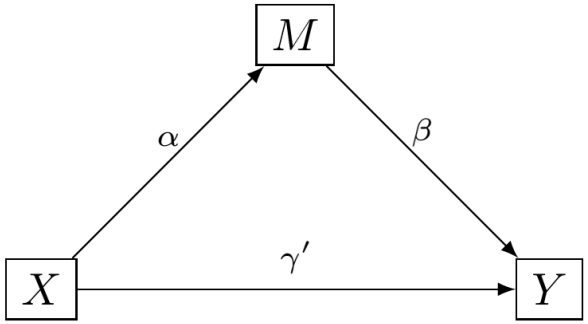
- Application of these rules yields 27 models





Expected mediated effects

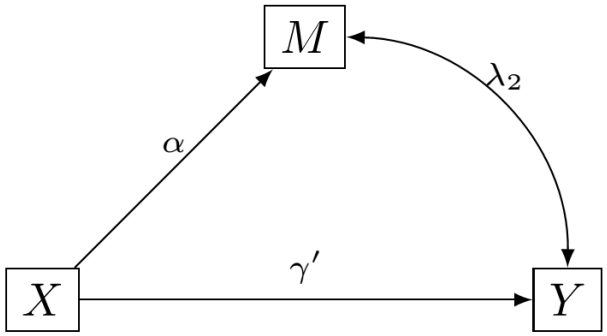
- We have also derived the *true* mediated effect from X to Y via M in each of these models and the *expected* mediated effect if the model is (incorrectly) analyzed, assuming that the X→ M → Y model holds



$$\rho_{XM} = \alpha$$
$$\rho_{MY} = \beta + \alpha\gamma'$$
$$\rho_{XY} = \gamma' + \alpha\beta$$

$$\beta_{MY.X} = \frac{(\beta+\alpha\gamma')-\alpha(\gamma'+\alpha\beta)}{1-\alpha^2}$$
$$\beta_{MY.X} = \frac{\beta+\alpha\gamma'-\alpha\gamma'-\alpha^2\beta}{1-\alpha^2}$$
$$\beta_{MY.X} = \frac{\beta-\alpha^2\beta}{1-\alpha^2}$$
$$\beta_{MY.X} = \frac{\beta(1-\alpha^2)}{1-\alpha^2}$$
$$\beta_{MY.X} = \beta$$

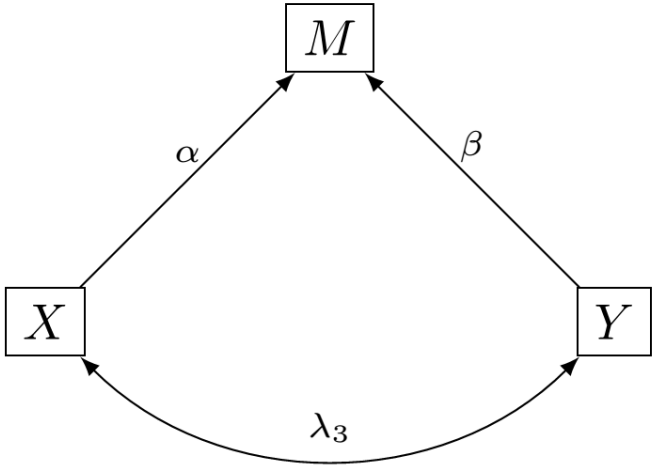
$$\rho_{XM} \times \beta_{MY.X} = \alpha\beta$$



$$\rho_{XM} = \alpha$$
$$\rho_{MY} = \lambda_2 + \alpha\gamma'$$
$$\rho_{XY} = \gamma'$$

$$\beta_{MY.X} = \frac{(\lambda_2+\alpha\gamma')-\alpha\gamma'}{1-\alpha^2}$$
$$\beta_{MY.X} = \frac{\lambda_2}{1-\alpha^2}$$

$$\rho_{XM} \times \beta_{MY.X} = \frac{\alpha\lambda_2}{1-\alpha^2}$$



$$\rho_{XM} = \alpha + \lambda_3\beta$$
$$\rho_{MY} = \beta + \alpha\lambda_3$$
$$\rho_{XY} = \lambda_3$$

$$\beta_{MY.X} = \frac{(\beta+\alpha\lambda_3)-(\alpha+\lambda_3\beta)\lambda_3}{1-(\alpha+\lambda_3\beta)^2}$$
$$\beta_{MY.X} = \frac{\beta(\lambda_3^2-1)}{\alpha^2+2\alpha\beta\lambda_3+\beta^2\lambda_3^2-1}$$

$$\rho_{XM} \times \beta_{MY.X} = (\alpha + \lambda_3\beta) \times \frac{\beta(\lambda_3^2-1)}{\alpha^2+2\alpha\beta\lambda_3+\beta^2\lambda_3^2-1}$$
$$\rho_{XM} \times \beta_{MY.X} = \frac{\beta(\alpha+\lambda_3\beta)(\lambda_3^2-1)}{\alpha^2+2\alpha\beta\lambda_3+\beta^2\lambda_3^2-1}$$

Expected mediated effects

- Without the need to simulate we can derive what the *true* mediated effect will be and what the *expected* mediated effect under a wrong model would be

- We can e.g., easily show that it is perfectly plausible that an incorrect causal model can have a much larger indirect effect than the true causal model

Equivalence class

- How can we make the equivalence class smaller?

By imposing structural knowledge that is based on theory or research design

Narrowing down the equivalence class

- Was X randomized?
  - No arrows into X
- Is X temporally / causally before M?
  - No directed arrows from M to X
- Is X temporally / causally before Y?
  - No directed arrows from Y to X
- Is M temporally / causally before Y?
  - No directed arrows from Y to M

Narrowing down the equivalence class

- Definitive answers (i.e., not answering “don’t know”) will reduce the equivalence class – sometimes dramatically

- Answering these four simple questions results in potentially 54 unique situations in which a smaller number of equivalence models are plausible

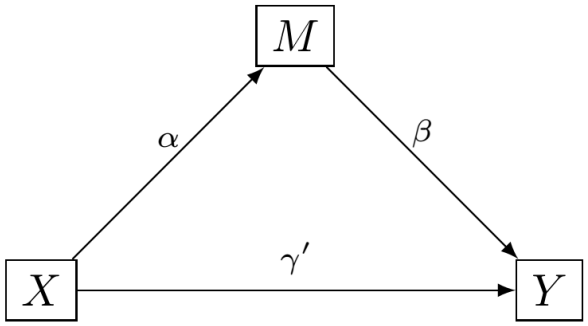
Equivalence class

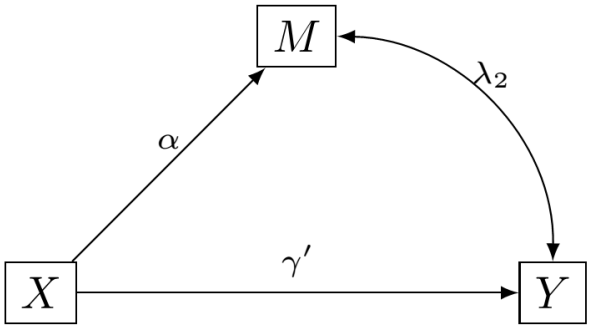
- X randomized?
  - No → do not exclude any models
- X before M?
  - Yes → exclude all models in which paths go from M into Y
- M before Y?
  - Unknown → do not exclude any models
- X before Y?
  - Yes → exclude all models in which paths go from Y to X
- Equivalent models: 12

Equivalence class

- We have generated complete “decision trees” and which equivalent models are yielded given assumed knowledge

	A	B	C	D	E
1		X randomizec	X before M	X before Y	M before Y
2	1	YES	YES	YES	YES
3	2	YES	YES	YES	NO
4	3	YES	YES	YES	DK
5	4	YES	YES	NO	YES
6	5	YES	YES	NO	NO
7	6	YES	YES	NO	DK
8	7	YES	YES	DK	YES
9	8	YES	YES	DK	NO
10	9	YES	YES	DK	DK
11	10	YES	NO	YES	YES
12	11	YES	NO	YES	NO
13	12	YES	NO	YES	DK
14	13	YES	NO	NO	YES
15	14	YES	NO	NO	NO
16	15	YES	NO	NO	DK
17	16	YES	NO	DK	YES
18	17	YES	NO	DK	NO
19	18	YES	NO	DK	DK
20	19	YES	DK	YES	YES
21	20	YES	DK	YES	NO
22	21	YES	DK	YES	DK
23	22	YES	DK	NO	YES
24	23	YES	DK	NO	NO
25	24	YES	DK	NO	DK
26	25	YES	DK	DK	YES
27	26	YES	DK	DK	NO
28	27	YES	DK	DK	DK
29	28	NO	YES	YES	YES
30	29	NO	YES	YES	NO
31	30	NO	YES	YES	DK
32	31	NO	YES	NO	YES
33	32	NO	YES	NO	NO
34	33	NO	YES	NO	DK
35	34	NO	YES	DK	YES
36	35	NO	YES	DK	NO
37	36	NO	YES	DK	DK
38	37	NO	NO	YES	YES
39	38	NO	NO	YES	NO
40	39	NO	NO	YES	DK
41	40	NO	NO	NO	YES
42	41	NO	NO	NO	NO
43	42	NO	NO	NO	DK
44	43	NO	NO	DK	YES
45	44	NO	NO	DK	NO
46	45	NO	NO	DK	DK
47	46	NO	DK	YES	YES
48	47	NO	DK	YES	NO
49	48	NO	DK	YES	DK
50	49	NO	DK	NO	YES
51	50	NO	DK	NO	NO
52	51	NO	DK	NO	DK
53	52	NO	DK	DK	YES
54	53	NO	DK	DK	NO
55	54	NO	DK	DK	DK


M
α
β
γ′
X
Y


M
λ2
α
γ′
X
Y

Discussion

Contact:
felix.thoemmes@cornell.edu

Markov equivalence class

- In Markovian models, equivalence is guaranteed, if and only if, the same conditional independencies are implied by the models being compared

- This also means that for Markovian models, equivalence holds if graphs have the same “skeleton”, and “v-structure”

Equivalence class

- *Assuming presence of confounding*, one can identify at least 125 “prototypical” models (almost all of them under-identified)