Unipolar Item Response Models

Joseph F. Lucke

Research Institute on Addictions
State University of New York at Buffalo, NY

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Background

- Since 2003, item response theory (IRT) models have been increasing applied to addiction research.
- To date the number of studies has been
  - Alcohol: 12 studies (Keyes, Krueger, Grant, & Hasin, 2011);
  - Nicotine: 5 studies (Liu, Hedeker, & Mermelstein, 2013);
  - Illicit drug use: 9 studies (Saha et al., 2012); and
  - Gambling: 7 studies (Sharp et al., 2012).
- All of these studies have been concerned with item analyses of measures of addictive disorders.
- Addictive disorder as a unidimensional continuum rather than the traditional categories of “use”, “dependence”, “abuse”, and “addiction”.

Bipolarity Assumption

- Current IRT models assume the latent trait θ can occupy the entire real line, i.e., $-\infty < \theta < \infty$.
- Bipolarity assumption is appropriate for traits such as ability or attitude.
- Assumption creates both conceptual and technical problems traits such as addiction.

Measurement Problems

- There is no addiction level less than “no disorder”.
- The bipolarity assumption forces “no disorder” to be set $\theta = -\infty$.
- It also forces a non-addict to be infinitely distant from anyone with a addiction disorder, no matter how minor.
- If θ is a random variable, then usually $\Pr(\theta = -\infty) = 0$.
- Non-addicts are effectively excluded from the addiction continuum.

Theories of Addiction

- An addictive disorder is “a chronic condition of the motivational system in which a reward-seeking behavior [is] out of control” (West, 2006, p. 174).
- The chronic condition is caused by (Orford, 2001)
  - *cumulative ampliative effects* of an underlying reward system and
  - *inadequately damped* by eroded motivational constraints.
- Addiction is a *unipolar* disorder.
- A more realistic assumption is $0 \leq \theta < \infty$.
- Also, $\Pr(\theta \geq 0) \geq 0$, but $\Pr(\theta < 0) = 0$.

Informal Ideas for Unipolar IRM

- IRT models can more realistically represent the underlying psychological processes that may explain an individual's response to items (De Boeck & Wilson, 2004; van der Maas, Molenaar, Maris, Kievit, & Borsboom, 2011).
- Conceptualize the level of use disorder as a nonnegative latent trait.
- Model the cumulative ampliative and dampening effects as multiplicative processes.
- Implement a psychometric model based on Stevens's (1957) psychophysical power law (Thomas, 1983).

Further Development

- Let y_{ik} be a Bernoulli random variable such that $y_{ik} = 1$ if person i endorses item k and zero otherwise.
- Consider the psychophysical model: $R = \alpha S^{\beta}$.
- Replace R with the expected response $E(y_{ik})$.
- Replace S with latent trait $\theta_i \geq 0$ for person i.
- Reinterpret $\alpha_k > 0$ and $\beta_k > 0$ as scale and shape parameters for item k.
- Now we have $0 \leq E\left(y_{ik}\right) \leq 1$ and $0 \leq \alpha_k \theta_i^{\beta_k} < \infty$.
- Specify a link function G so that $0 \leq G\left[E\left(y_{ik}\right)\right] < \infty$.
- Then

$$G\left[E\left(y_{ik}\right)\right] = G\left[\Pr\left(y_{ik} = 1\right)\right] = \alpha_k \theta_i^{\beta_k}.$$

The Log-Logistic Unipolar Model

- Consider the *odds* link function:

$$G\left[\Pr\left(y_{ik}=1\right)\right]=\frac{\Pr(y_{ik}=1)}{\Pr(y_{ik}=0)}=\alpha_k\theta_i^{\beta_k}.$$

- This yields the log-logistic UIRM:

$$\pi_k(\theta_i)=\Pr\left(y_{ik}=1\,|\,\theta_i,\alpha_k,\beta_k\right)=\frac{\alpha_k\theta_i^{\beta_k}}{1+\alpha_k\theta_i^{\beta_k}}.$$

- Unipolar analogue to the canonical bipolar item response model.
- For $\beta_k=1$, this is Rasch's (1966) original item response model.

Additional Derivations

- Define the *severity* of an item by

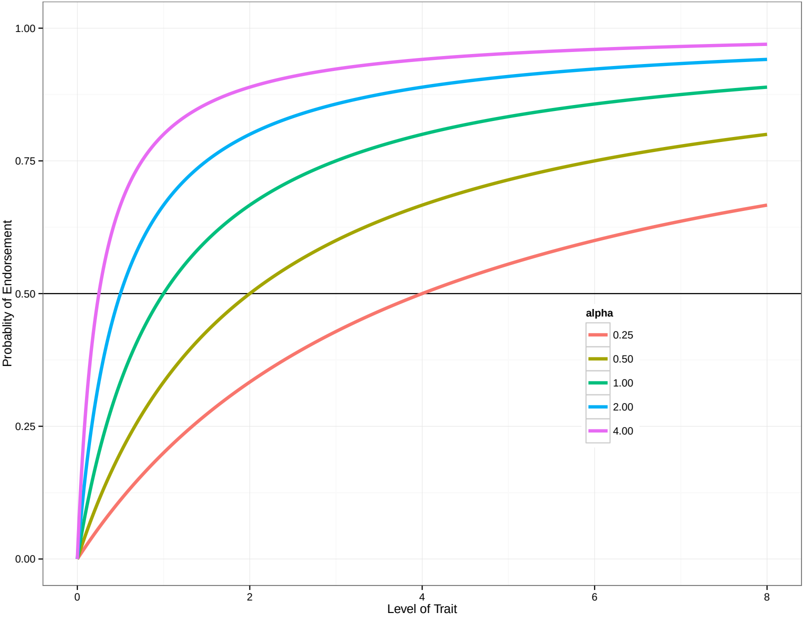
$$\delta_k = \left(\frac{1}{\alpha_k}\right)^{\frac{1}{\beta_k}}.$$

- δ_k is the point on the θ axis such that $\pi_k(\delta_k) = .5$.

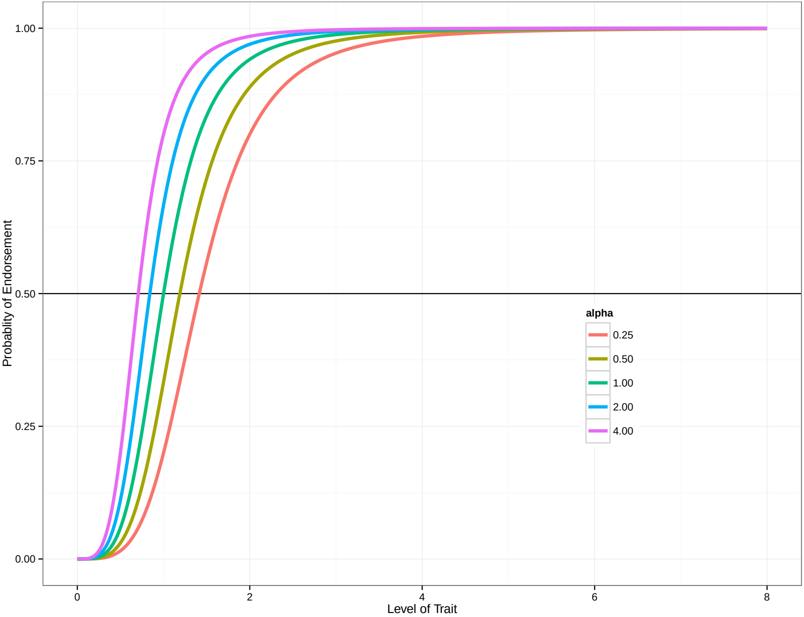
- The *item information function* (IIC) is the precision of an item as a function of θ.
- The log-logistic IIC for item k is

$$I_k(\theta) = \left(\frac{\beta_k}{\theta}\right)^2 \pi_k(\theta)\left[1 - \pi_k(\theta)\right].$$

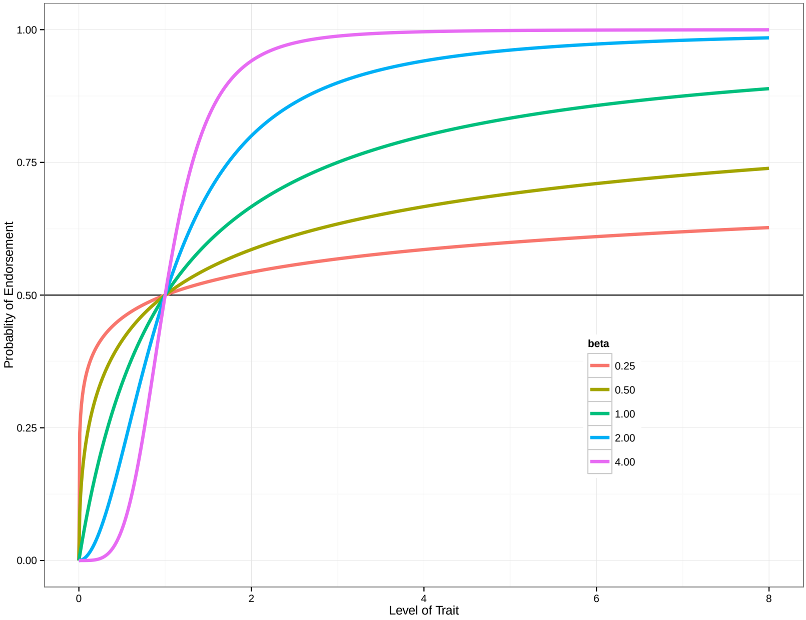
Log-Logistic UIRM: Varying α, $\beta = 1$



Log-Logistic UIRM: Varying α, $\beta = 4$



Log-Logistic UIRM: Varying β, $\alpha = 1$



Bayesian Inference

- Let I be the number of persons and K be the number of items.
- Assume independence among subjects and local independence among items, no missing data, prior independence among the parameters.
- The joint posterior density of the model parameters given the data is

$$p\left(\alpha_1, \ldots, \alpha_K; \beta_1, \ldots, \beta_K; \theta_1, \ldots, \theta_I \mid y_{11}, \ldots, y_{IK}\right) \propto$$
$$\prod_{i=1}^{I} \left\{ \prod_{k=1}^{K} \left[\alpha_k \theta_i^{\beta_k}\right]^{y_{ik}} \left[1 + \alpha_k \theta_i^{\beta_k}\right]^{-1} p(\alpha_k) p(\beta_k) \right\} p(\theta_i),$$

where $p(\alpha_k)$, $p(\beta_k)$, and $p(\theta_i)$ are the prior densities.

Markov Chain Monte Carlo: Gibbs Sampling

- Marginal parameter distributions were obtained by Gibbs sampling.
- Gibbs sampling theory IRT models has been thoroughly studied. (Curtis, 2010; Fox, 2010; Patz & Junker, 1999).
- Possessing marginal distributions allow one to compute any statistic regarding the parameter.
- The Bayesian approach allows the responses of all respondents to be used, including those who endorse no items and those who endorse all items.

Software

- All data management, analyses, and graphical displays were conducted in R (R Core Team, 2014) using Rstudio (RStudio, Inc, 2013).
- The parameters were estimated by MCMC using **JAGS** (Plummer, 2003, 2011) and the **R2jags** package (Su & Yajima, 2012).
- The graphics were produce with the **ggplot2** package (Wickham, 2009).

Data Set

- Two public-use files from the Clinical Trials Network for methadone and non-methadone maintenance trials (Peirce et al., 2006; Petry et al., 2005; Wu et al., 2009).
- $I = 687$
- $K = 7$ items from the DSM-IV.
- Of the $2^7 = 128$ possible response patterns, only 70 were represented.
- 415 of 687 (60.4%) had the response pattern of all 0's.
- 69 (10.0%) had the response pattern of all 1's.
- Remaining 203 (29.5%) roughly uniform distribution of 0's and 1's.

DSM Items

Item	Count	Content
DSM1	190	Increasing tolerance of alcohol
DSM2	118	Manifesting symptoms of alcohol withdrawal
DSM3	166	Drinking in large amounts or for longer periods
DSM4	210	Persistent desire to control use
DSM5	144	Spend large amounts of time in acquiring alcohol
DSM6	148	Forego important activities in order to drink
DSM7	176	Continued to drink despite persistent problems

Prior Densities

- $I = 687$ persons, $K = 7$ items, deviance statistic, so $2K + I + 1 = 702$ parameters.

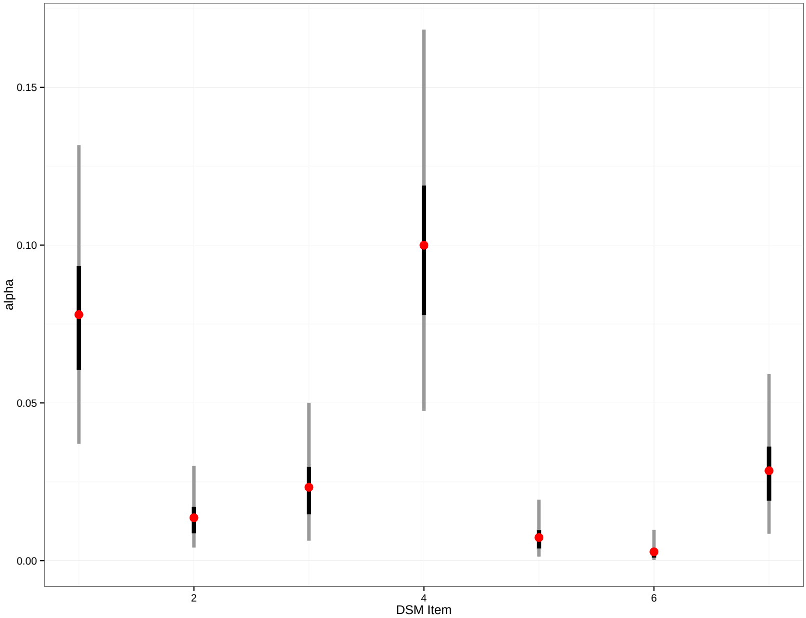
- If latent trait θ is generated by infinitesimal multiplicative effects, then θ will have asymptotically standard lognormal density (Johnson, Kotz, & Balakrishnan, 1994).
- $\theta_i \sim$ lognormal$(0, 1)$, so that $\Pr(0 < \theta_i < 5.2) = .95$.

- Low-information prior densities for the α's and β's.
- $\alpha_k \sim$ lognormal$(0, 10)$, so that $\Pr(0 < \alpha_k < 181.5) = .95$.
- If $\alpha_k \sim$ lognormal$(0, 10)$,, then $1/\alpha_k \sim$ lognormal$(0, 10)$.
- $\beta_k \sim$ gamma$(.1, .1)$, so that $\Pr(0 < \beta_k < 5.8) = .95$

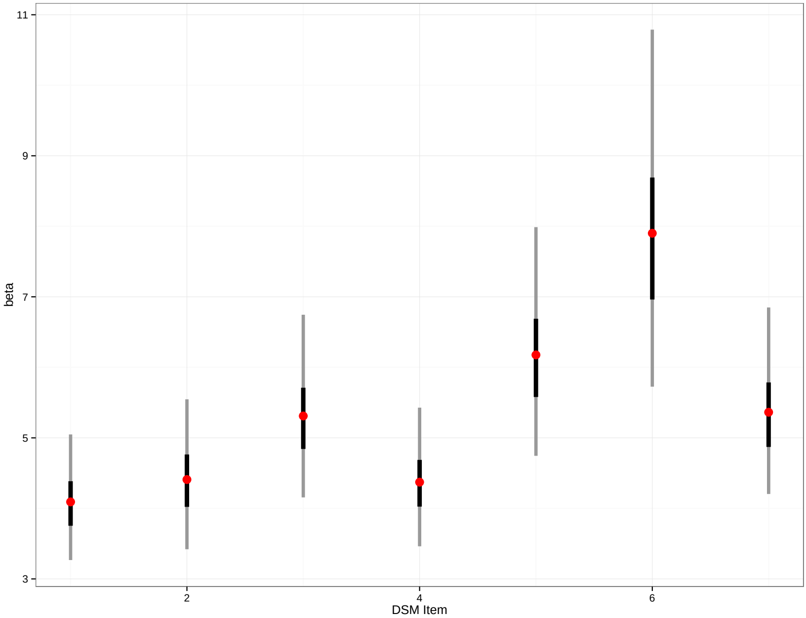
MCMC Procedure

- Initial run: 1,000 simulations with 3 chains.
- Final run: 10,000 simulations thinned by 10s, 3 chains.
- All MCMC convergence diagnostics were satisfactory
- Brooks-Gelman-Rubin $1.00 < \hat{R} < 1.01$ for all parameters
- Effective sample sizes were 340 – 3000 for all parameters.
- Autocorrelations were effectively zero for all non-zero lags.
- DIC $= 2277.76$.

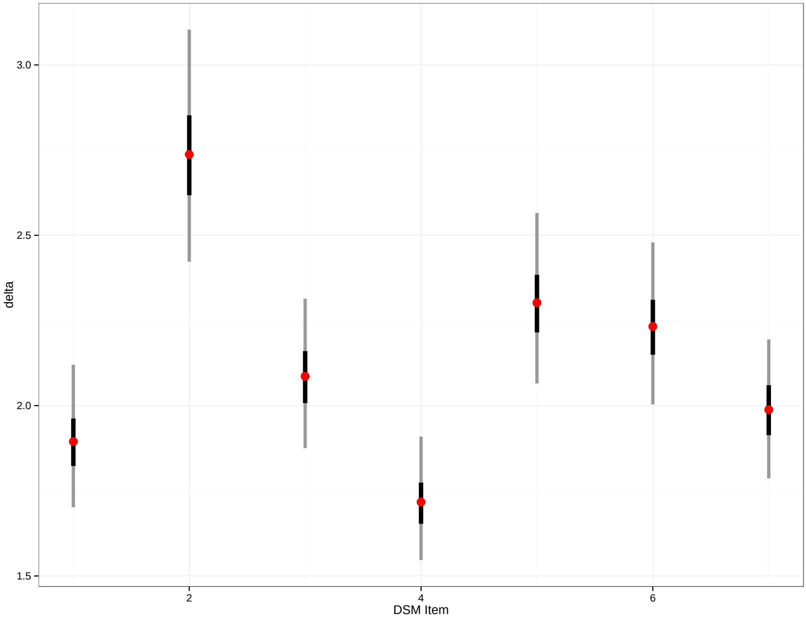
Expected α's



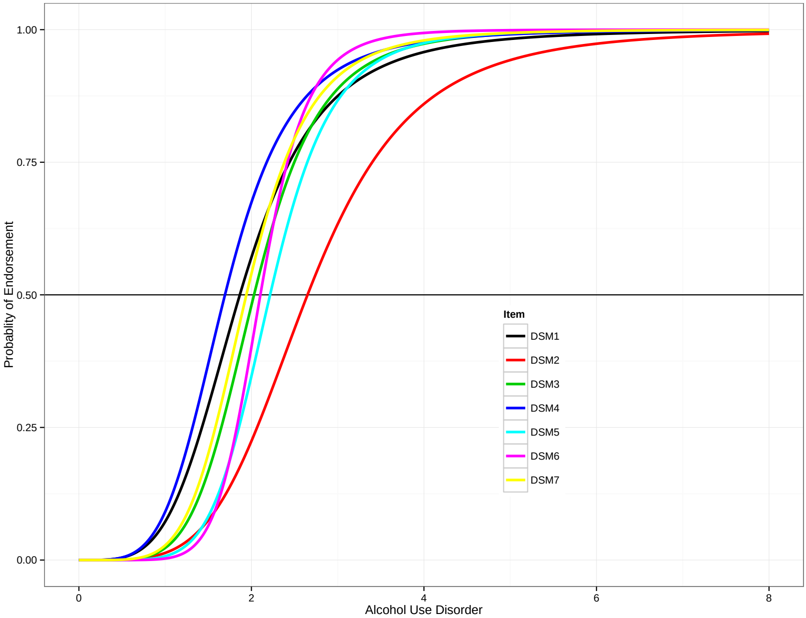
Expected β's



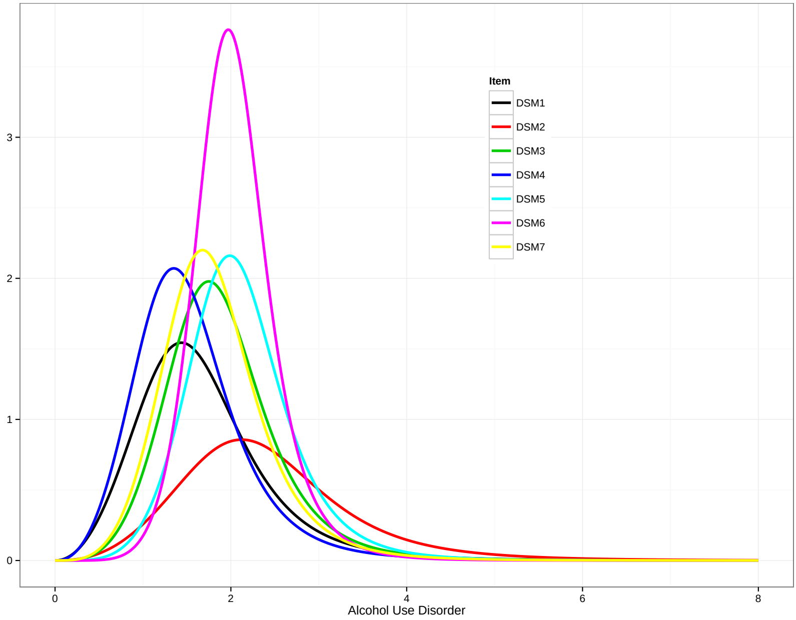
Expected Severities, δ's



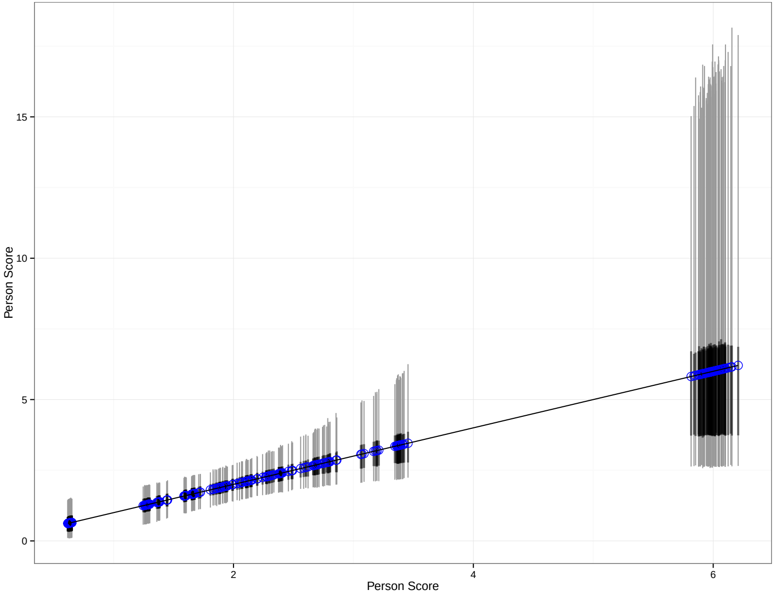
Estimated Item Characteristic Curves



Estimated Item Information Curves



Posterior Person Scores



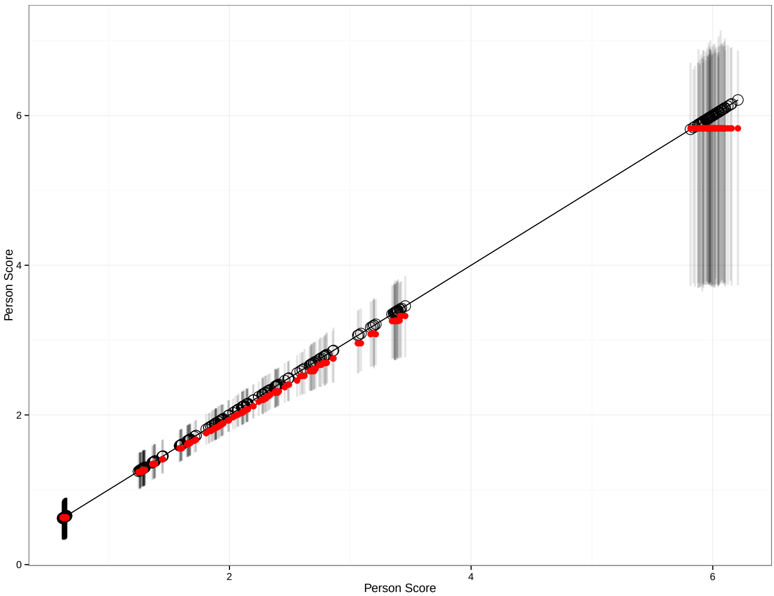
Person Scores

- A crucial goal in IRT analysis is to estimate the person score θ given the item responses $\mathbf{y}$.
- Here I obtain $E(\theta \mid \mathbf{y})$ using an empirical Bayes approach.
- Let $\alpha_k^* = E(\alpha_k \mid \mathbf{y})$ and $\beta_k^* = E(\beta_k \mid \mathbf{y})$ for all k.
- Then

$$E(\theta \mid \mathbf{y}) = \frac{\int_0^\infty \prod_{k=1}^K \theta \left[\alpha_k^* \theta^{\beta_k^*}\right]^{y_{ik}} \left[1 + \alpha_k^* \theta^{\beta_k^*}\right]^{-1} p(\theta) d\theta}{\int_0^\infty \prod_{k=1}^K \left[\alpha_k^* \theta^{\beta_k^*}\right]^{y_{ik}} \left[1 + \alpha_k^* \theta^{\beta_k^*}\right]^{-1} p(\theta) d\theta}.$$

- Recall that $\theta \sim$ lognormal$(0, 1)$.

Estimated Person Scores



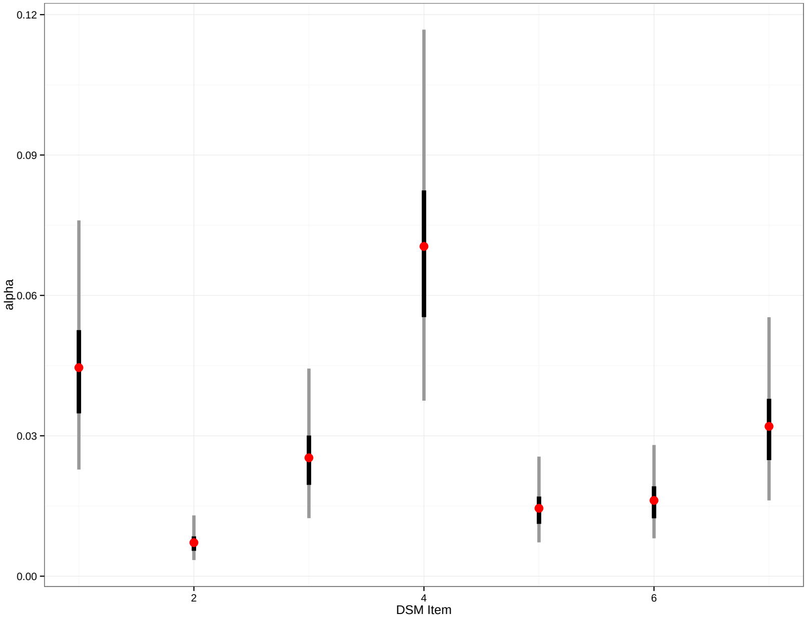
One-Parameter Model

- Setting $\beta_k = \beta$ yields

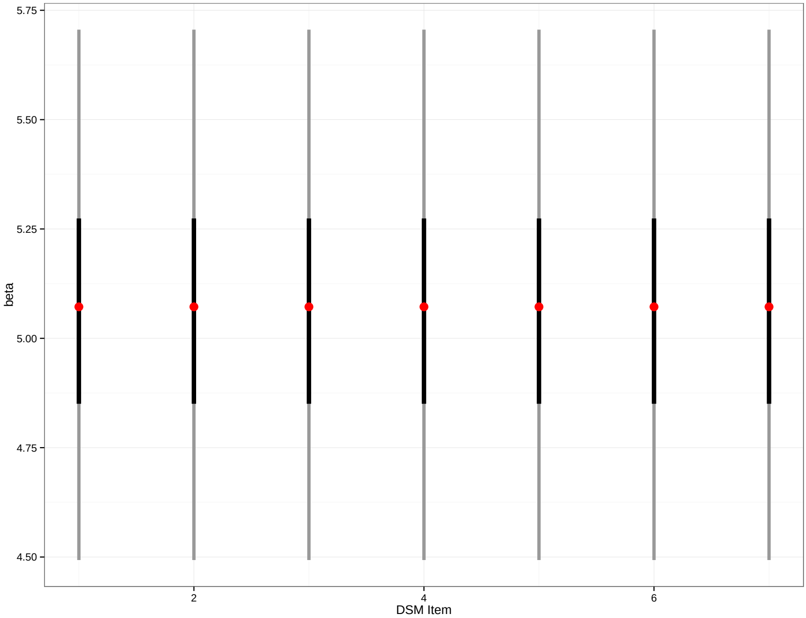
$$\pi_k(\theta_i) = \Pr(y_{ik} = 1 \mid \theta_i, \alpha_k, \beta) = \frac{\alpha_k \theta_i^{\beta}}{1 + \alpha_k \theta_i^{\beta}}.$$

- Replace 7 parameters with 1.
- Same MCMC procedure.
- All convergence diagnostics the same or slightly better.
- DIC $= 2260.05$.

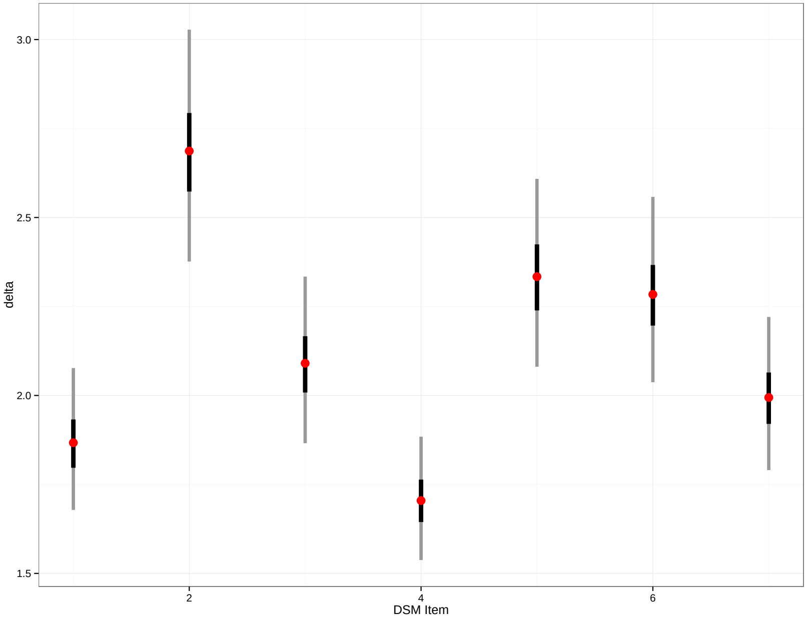
Estimated α's



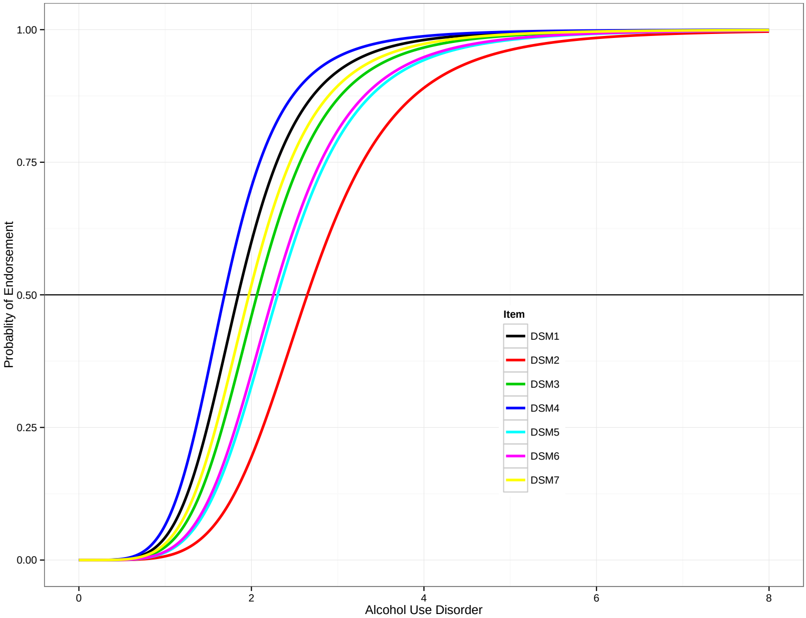
Estimated β's



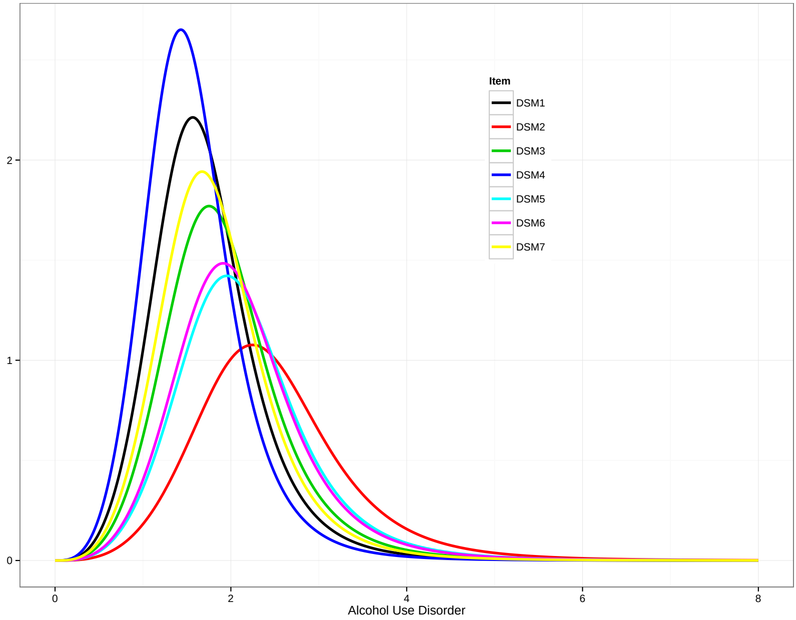
Estimated δ's



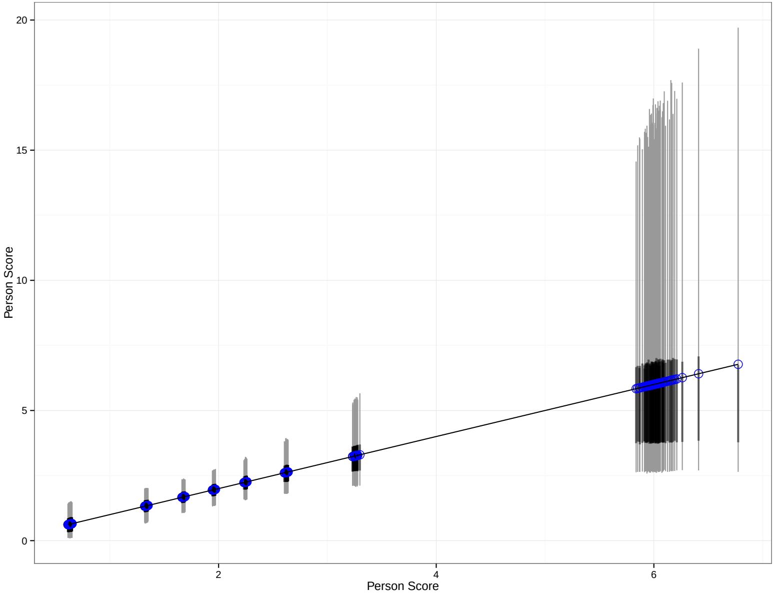
Item Characteristic Curve



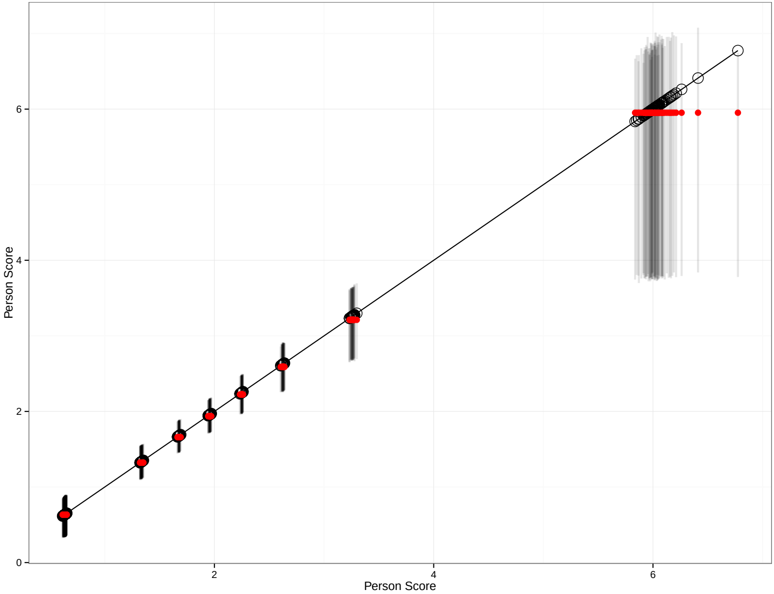
Item Information Curve



Posterior Person Scores



Estimated Person Scores



Unipolar Models Are Not New.

- The original Rasch model posited a unipolar trait for the subject's ability (1966, pp. 90–91).
- Reise and Waller (2009, pp. 29–32) argued that standard (bipolar) IRT models of clinical phenomena, such as depression, were possibly misrepresenting unipolar traits.
- van der Maas et al. (2011) proposed a *positive ability model* derived from cognitive information processing principles.
- Lucke (2014) proposed the *positive trait item response model*.

A General Unipolar Item Response Model

- First, extend the response model (Chastrette, Thomas-Danguin, & Rallet, 1998).
- Let γ_k be *baseline rate* of response to item k for a person i with no disorder, $\theta_i = 0$.
- Thus, $E(y_{ik}) = \gamma_k + \alpha_k \theta_i^{\beta_k}$.

- Next, allow a wider class of link functions.
- Let F be any absolutely continuous distribution function with positive support.
- Choice of F yields different models.

- Then a general UIRM is

$$\pi_k(\theta_i) = \Pr(y_{ik} = 1 \mid \theta_i, \gamma_k, \alpha_k, \beta_k) = F\left(\gamma_k + \alpha_k \theta_i^{\beta_k}\right).$$

Four Specific Models

- The log-logistic:

$$\pi_{\mathsf{L}}(\theta) = \frac{\gamma + \alpha\theta^{\beta}}{1 + \gamma + \alpha\theta^{\beta}}.$$

- The log-normal:

$$\pi_{\mathsf{N}}(\theta) = \Phi\left[\log\left(\gamma + \alpha\theta^{\beta}\right)\right].$$

- The Weibull:

$$\pi_{\mathsf{W}}(\theta) = 1 - \exp\left(-\gamma - \alpha\theta^{\beta}\right).$$

- The log-Cauchy:

$$\pi_{\mathsf{C}} = \frac{1}{2} + \frac{1}{\pi}\arctan\left[\log\left(\gamma + \alpha\theta^{\beta}\right)\right].$$

Extensions

- Missing data.
- Ordinal and nominal responses.
- Fixed and random item effects.
- Fixed and random person effects.
- Multidimensional person parameters.
- Priors for latent traits.

Summary

- UIRMs appear to be a viable alternative to the usual IRT models for positive traits.
- Bayesian inference via MCMC is satisfactory for obtaining item parameter and person parameter distributions.
- Substantial flexibility in UIRMs is available.
- UIRMs show the general flexibility of IRT to realistically model the relations between items and persons.

References I

Chastrette, M., Thomas-Danguin, T., & Rallet, E. (1998). Modelling the human olfactory stimulus-response function. *Chemical Senses*, *23*, 181–196.

Curtis, S. M. (2010 31). BUGS code for item response theory. *Journal of Statistical Software*, *36*(1), 1–34.

De Boeck, P., & Wilson, M. (2004). *Explanatory item response models: A generalized linear and nonlinear approach*. New York, NY: Springer.

Fox, J.-P. (2010). *Bayesian item response modeling: theory and applications*. New York, NY: Springer.

Johnson, N. L., Kotz, S., & Balakrishnan, N. (1994). *Continuous univariate distributions.* (2nd ed., Vol. 1). New York, NY: Wiley.

Keyes, K. M., Krueger, R. F., Grant, B. F., & Hasin, D. S. (2011). Alcohol craving and the dimensionality of alcohol disorders. *Psychological Medicine*, *41*(03), 629-640.

Liu, L. C., Hedeker, D., & Mermelstein, R. J. (2013). Modeling nicotine dependence: An application of a longitudinal IRT model for the analysis of Adolescent Nicotine Dependence Syndrome Scale. *Nicotine & Tobacco Research*, *15*(2), 326-333.

Lucke, J. F. (2014). Positive trait item response models. In R. E. Millsap, L. A. van der Ark, D. M. Bolt, & C. M. Woods (Eds.), *New developments in quantitative psychology : Presentations from the 77th annual psychometric society meeting.* New York: Springer.

Orford, J. (2001). *Excessive appetites: a psychological view of addictions* (2nd ed.). Chichester, UK: Wiley.

Patz, R. J., & Junker, B. W. (1999). A straightforward approach to Markov chain Monte Carlo methods for item response models. *Journal of Educational and Behavioral Statistics*, *24*(2), 146-178.

Peirce, J. M., Petry, N. M., Stitzer, M. L., Blaine, J., Kellogg, S., Satterfield, F., et al. (2006). Effects of lower-cost incentives on stimulant abstinence in methadone maintenance treatment: a National Drug Abuse Treatment Clinical Trials Network study. *Archives of General Psychiatry*, *63*(2), 201-208.

Petry, N. M., Peirce, J. M., Stitzer, M. L., Blaine, J., Roll, J. M., Cohen, A., et al. (2005). Effect of prize-based incentives on outcomes in stimulant abusers in outpatient psychosocial treatment programs: a National Drug Abuse Treatment Clinical Trials Network study. *Archives of General Psychiatry*, *62*(10), 1148-1156.

Plummer, M. (2003). JAGS: A program for analysis of Bayesian graphical models using Gibbs sampling. In K. Hornik, F. Leisch, & A. Zeileis (Eds.), *Proceedings of the 3rd international workshop on distributed statistical computing.* Vienna, Austria: Technische Universität Wien. (Available at `http://sourceforge.net/projects/mcmc-jags/`)

Plummer, M. (2011). *JAGS version 3.1.0 user manual.* (Available at `http://sourceforge.net/projects/mcmc-jags/files/Manuals/3.x/jags_user_manual.pdf/download`)

References II

R Core Team. (2014). *R: A language and environment for statistical computing.* Vienna, Austria. (Available at `http://cran.us.r-project.org/`)

Rasch, G. (1966). An individualistic approach to item analysis. In P. F. Lazarsfeld & N. W. Henry (Eds.), *Readings in mathematical social science* (pp. 89–108.). Chicago, IL: Chicago Science Research Associates.

Reise, S. P., & Waller, N. G. (2009). Item response theory and clinical measurement. *Annual Review of Clinical Psychology*, *5*(1), 27-48. (PMID: 18976138)

RStudio, Inc. (2013). *Rstudio.* Boston, MA. (Available at `www.rstudio.com/`)

Saha, T. D., Compton, W. M., Chou, S. P., Smith, S., Ruan, W. J., Huang, B., et al. (2012). Analyses related to the development of DSM-5 criteria for substance use related disorders: 1. Toward amphetamine, cocaine and prescription drug use disorder continua using item response theory. *Drug and Alcohol Dependence*, *122*(1-2), 38–46.

Sharp, C., Steinberg, L., Yaroslavsky, I., Hofmeyr, A., Dellis, A., Ross, D., et al. (2012). An item response theory analysis of the Problem Gambling Severity Index. *Assessment*, *19*(2), 167-175.

Stevens, S. S. (1957). On the psychophysical law. *Psychological Review*, *64*(3), 153 - 181.

Su, Y.-S., & Yajima, M. (2012). *R2jags: A package for running JAGS from R.* (Available at `http://cran.r-project.org/web/packages/R2jags/R2jags.pdf`)

Thomas, H. (1983). Parameter estimation in simple psychophysical models. *Psychological Bulletin*, *93*(2), 396–403.

van der Maas, H. L. J., Molenaar, D., Maris, G., Kievit, R. A., & Borsboom, D. (2011). Cognitive psychology meets psychometric theory: On the relation between process models for decision making and latent variable models for individual differences. *Psychological Review*, *118*(2), 339 - 356.

West, R. (2006). *Theory of addiction*. Oxford, UK: Blackwell.

Wickham, H. (2009). *ggplot2: Elegant graphics for data analysis*. New York, NY: Springer.

Wu, L.-T., Pan, J.-J., Blazer, D. G., Tai, B., Stitzer, M. L., Brooner, R. K., et al. (2009). An item response theory modeling of alcohol and marijuana dependences: a National Drug Abuse Treatment Clinical Trials Network study. *Journal of Studies on Alcohol and Drugs*, *70*(3), 414–425.